Modeling the Impedance of Water-Cooled Core-Less Multi-Layered Solenoid Coils for MPI Drive Field Generation

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Abstract

A complete lumped component model representing the wideband impedance of water-cooled multi-layered core-less solenoid coils is presented and analytical and numerical calculation methods for model elements are reviewed and extended. The model includes stray capacitances, mutual inductances and frequency-dependent resistive losses. Contrary to previous treatments of this topic, the model is not simplified further and is evaluated in its complete form, allowing accurate prediction of the coil impedance beyond the first resonant frequency. This aspect is especially important if the coil is part of a passive filter circuit, where higher resonances limit the filter bandwidth. Additionally, a liquid coolant is included in the calculations. Additionally, figures of merit for the evaluation of field homogeneity inside the coil are given. The model is applied to a MPI drive coil and is compared to measured data. It shows good agreement up to 4 MHz, including the second series resonance of the coil. Additionally, the influence of water-cooling on the coil impedance is investigated. Comparison of model results to measured data shows additional losses.

I. Introduction

Parasitic effects have a large impact on the high frequency behavior of coils. From the first resonant frequency onward, the coil impedance is dominated by the interaction of parasitic capacitances, mutual- and self inductances of individual turns, as well as frequency-dependent loss mechanisms such as skin- and proximity effect.

For the optimization of passive filter circuits, coil behavior beyond the first resonant frequency is important, because (higher) resonances often limit the bandwidth over which the filter maintains the desired response. Since Magnetic Particle Imaging (MPI) drive field coils, which are used for field generation in the kHz range, are always a part of the transmit filters, these resonances are an important design consideration. This influence on the filter bandwidth happens because either the first (parallel) resonance or the second (series) resonance of the coil introduces an undesired pair of poles or zeros (respectively) into the coil’s impedance, leading to deviations from its ideal characteristic. Since common filter synthesis assumes that coils only exhibit an impedance zero at the origin, the response of the filter deviates from the intended transfer function. For the bandpass and bandstop filters used in MPI, this results in limited attenuation at high frequencies or a limited maximum pass-band range respectively. Prediction of coil parameters before construction can therefore reveal whether these effects will limit scanner performance and...
if a different construction or filter synthesis is advised. Previous publications have established a lumped component coil model [5, 6] and provided a procedure to iteratively simplify it to extract the total capacitance value [7]. Additionally, numerical [8] and analytical [5–7] methods for the calculation of stray capacitances have been suggested. For electromagnetic compatibility (EMC) applications, the partial element equivalent circuits (PEEC) method [9] shows good agreement with measured data [10]. Although it provides good insight into the problem, field solvers using this method are not generally available for non-planar geometries. Also, to the knowledge of the authors, it has not been demonstrated for multi-layered coils or coils in dielectric media other than air. From a design perspective, it can be beneficial to let simulation elements coincide with the elements of the designed geometry, so that cell results directly correspond to design parameters.

This paper presents a method to predict the impedance and homogeneity of a multi-layered coreless solenoid coil over a wide frequency range. Using analytical formulas or finite element method (FEM) data in combination with a SPICE circuit simulator [11]. The simplifications compared to a full PEEC simulation in combination with analytic element estimation allow faster simulation times and good scaling, making it suitable for initial design verification and iterative optimization techniques. Unlike previous works, where individual circuit elements were removed from the network before solving it (e.g. the mutual inductance was added to the self-inductance [7]), the proposed method works beyond the first resonant frequency since it includes the interactions between individual windings. Additionally, we investigate the influence of water cooling (i.e. coil windings submerged in water) on the coil’s impedance and the self resonant frequency, which has not been addressed before. The method was used to study the behavior of a MPI drive coil. Comparison of simulation results with measured data shows good agreement up to the second series resonance.

II. Methods

Computational electromagnetic methods are routinely used nowadays for the design of coils. However, if parasitic properties of the coil need to be determined, the complexity of the required computation rises. If the size of the structure is well below the wavelengths at the frequency range of interest, a lumped component model can be assembled to represent the problem. After determining individual element values, the lumped model can be solved instead, which is usually less computational intensive than a (full wave) EM simulation. It can also be combined with other component models, e.g. to predict the behavior of complete filter structures. To adequately represent a coil up to and beyond the first resonant frequency, the mutual and self inductance of the windings, as well as stray capacitances between them need to be considered. Also, frequency-dependent losses need to be modeled. When determining element values, it can be beneficial to use analytical formulas, especially in early design phases when fast verification of design intent is more important than high accuracy.

It should be noted that at frequencies where the current paths are significantly altered by displacement currents, the modeling of the magnetic coil properties needs to take these into account. To continue using a lumped component model, a smaller cell size as well as additional inductances are required and the element values become frequency dependent, which is beyond the scope of this work. Another limit appears once the wavelengths are of the same order as the size of the structures, so that retarded fields need to be modeled. From that frequency onward, full wave or full PEEC simulations are more adequate. For the frequency range of interest in MPI drive field applications, which is in the 10 kHz to 150 kHz range, field retardation is not required for structures with dimensions below several hundred meters. Even for MPI receive circuitry, which uses frequencies up to ≈ 2 MHz, lumped components models are usually sufficient for the geometries encountered in typical MPI scanners. At these frequency ranges the presented method is easier to apply than full wave simulations that struggle with the large span of geometric feature sizes in MPI coils. It also gives more accurate results than first order models that cannot predict higher coil resonances.

II.I. RLC Model for Quantitative Comparison

Figure 1 shows a commonly used RLC model that represents the whole coil including its first (parallel) resonance. We will use it to compare the results of a more sophisticated model network with measured data quantitatively. It includes the coil’s inductance $L_S$, models its first parallel resonance through capacitance $C_p$, includes the coil’s DC resistance through series resistance $R_S$ (because usually $R_S \parallel R_0 \approx R_S$) and the quality factor of the resonance by introducing a parallel resistance $R_0$ (cf. [12]). Although higher order models are available, fitting these to measured data becomes increasingly ambiguous because several combinations of model parameters can yield the same residual sum of quadratic errors, which makes these higher order models unsuitable for parameter comparison.

II.II. Modeling Stray Capacitances Analytically

Several approaches have been presented to calculate the mutual capacitance of individual solenoid coil windings.
All techniques assume that the winding radius \( R \) of the coil is much larger than the wire radius \( r_{\text{cu}} \) and neglect the curvature. In the following we want to denote the turn-to-turn capacitance between neighboring wires of the same layer as \( C_t \) and distinguish it from the inter-layer capacitance \( C_l \) between windings of different layers.

Different assumptions have been made considering the effect of the opposite winding pitch of adjacent layers in a multi-layered coil: While Massarini et al. [5] assumed densely packed windings with a staggered pattern, Hole et al. [7] declared that adjacent layers will stack without shift due to the opposing pitch. Geometrical considerations suggest that both variants appear: since a densely wound coil has a pitch of \( 2r_{\text{cu}} \) per turn, adjacent layers with opposite pitch will show the same cross-sectional pattern after every 180°, so both patterns alternate every 90° (cf. Figure 2).

To calculate the capacitance value, we have expanded the method proposed by Massarini et al. [5] to support spacing between wires. This allows us to use this approach for both winding patterns and incorporate manufacturing tolerances. We also incorporated surrounding dielectrics other than air. Similar to the original derivation we consider the series connection formed by the wire and the insulating coating with the ambient medium to another insulated wire. In the case where litz wire is used, it assumes that the thickness of the wire serving and the gap size between the wires are large compared to size of individual wire strands and therefore the deviations from the round shape can be neglected. Due to the symmetric nature of the arrangement, it is sufficient to calculate the capacitance of one half and account for the series arrangement. Figure 3 shows two adjacent windings with the associated dimensions and material properties.

The capacitance per differential angle of the series connected cylindrical capacitors formed by the wires and their insulation can be calculated through a conformal mapping as [5]

\[
dC_m = \frac{\varepsilon_\text{in} l_1}{2 \ln \left( \frac{r_1}{r_0} \right)} d\theta, \tag{1}
\]

where \( l_1 = 2\pi R \) is the turn length, \( r_{\text{cu}} \) is the wire radius, \( \varepsilon_\text{in} \) is the dielectric constant for the insulation material and \( r_m \) its outer radius.

\( C_m \) is in series with the capacitance \( C_g \) of the gap between two wires. Following the simplifications of Massarini et al., we assume that the electric field lines form straight connections between the wires. This assumption is adequate if the angular segment is small and the wires are densely packed. For each differential angle, using the basic capacitor formula with the differential surface element \( dS \), we arrive at

\[
dC_g = \varepsilon_g \frac{dS}{2 x(\theta)} = \frac{\varepsilon_g l_1}{2} \frac{r_m}{r_g r_m (1 - \cos \theta)} d\theta. \tag{2}
\]

Here, \( \varepsilon_g \) is the dielectric constant of the space between the wires, which are separated by a distance of \( 2r_g \). The expression for \( x(\theta) \) follows from the geometry shown in Figure 3.
The total capacitance follows from the series connection of the segments.

\[
    dC_{ser} = \frac{dC_{in}}{dC_{in} + dC_{g}} = \frac{\varepsilon_l \varepsilon_0 \varepsilon_{in} \arctan \left( \frac{(s_0 + 2r_0) s_{in}}{r_{in}} \right)}{S}. \tag{3}
\]

It includes the contributions of the insulations of both wires and the space between them. By integrating along the angle \( \theta \) we arrive at an expression for the capacitance between two adjacent windings

\[
    C_{ser} = \frac{l_n r_{in} \varepsilon_l \varepsilon_{in} \arctan \left( \frac{(s_0 + 2r_0) s_{in}}{r_{in}} \right)}{S}. \tag{4}
\]

\[
    S = \sqrt{(s_0 + 2r_0) s_{in}^2 + (s_0 + r_0)^2} \quad D = \varepsilon_l r_{in} \ln \left( \frac{r_{in}}{r_{cu}} \right). \tag{5}
\]

and evaluating this for an angle \( \theta = \pm \frac{\pi}{6} \) (cf. [5]) results in

\[
    C_{it} = 2C_{ser} \theta = \frac{2l_n r_{in} \varepsilon_l \varepsilon_{in} \arctan \left( \frac{r_{in} s_{in} + 2r_0 s_{in}}{r_{in} s_{in}} \right)}{S}. \tag{7}
\]

Since it includes spacing and an intermediate medium, the same formula was used to calculate the inter-layer capacitance \( C_t \).

### II.III. FEM Modeling of Capacitances

The capacitance between adjacent wires can be calculated numerically by using finite element methods. To calculate the coupling capacitances to all neighboring wires efficiently, the potential of a wire \( n \) is changed by a small value \( \Delta V_n \) compared to a reference simulation. For each adjacent wire \( m \), the charge variation \( \Delta Q_m \) is determined, while all wires except \( n \) are held at a constant potential. The individual coupling capacitances can be calculated by

\[
    C_{it,m,n} = -\frac{\Delta Q_m}{\Delta V_n} \bigg|_{V_n=\text{const}}. \tag{8}
\]

To get a quick result of the total capacitance of the coil that forms the first resonance with the coil’s inductance, the complete structure can be simulated at once. Each wire should be assigned with a potential according to its position in the winding process (e.g. by assuming a unit voltage drop \( V_{\text{coil}} \) over the coil and a constant voltage drop per winding). The capacitance can be calculated by evaluating the total electrostatic energy \( E_E \) in the structure according to

\[
    C = \frac{2E_E}{V_{\text{coil}}^2}. \tag{9}
\]

For the calculations in this work the open source FEMM (Finite Element Method Magnetics) package [13] has been used. Alternatively, many similar programs are available for this task.

### II.IV. Analytical Modeling of Self- and Mutual Inductances

To calculate the self-inductance of the wires we rely on the well-known formula [14, 15] for the inductance of a wire loop

\[
    L_m = \mu_0 R \left( \ln \left( \frac{8R}{r_{cu}} \right) - 2 + \frac{Y}{2} \right) \quad \tag{10}
\]

where \( \mu_0 \) is the vacuum permeability, \( R \) is the coil radius and \( Y \) is a parameter that accounts for the current distribution in the wire. It is 0 when assuming surface currents and \( \frac{1}{2} \) for a homogeneous current density across the wire cross section.

The mutual inductance between two coaxial wire loops \( m \) and \( n \) with the radii \( R_m, R_n \) and the positions \( z_m, z_n \) on the common axis, can be calculated using Maxwell’s formula [14, 16]

\[
    M_{m,n} = 4\pi \sqrt{R_m R_n} \left( \frac{2}{k} - k \right) E_k - \frac{2}{k} E_k \quad \tag{11}
\]

\[
    k = \frac{2\sqrt{R_m R_n}}{\sqrt{(R_m + R_n)^2 + (z_m - z_n)^2}} \quad \tag{12}
\]

where \( E_k \) and \( E_k \) are the complete elliptic integrals of the first and second kind respectively. It should be noted that many numerical implementations of these integrals expect \( k^2 \) as argument.

### II.V. FEM Modeling of Self- and Mutual Inductances

Similar to the capacitance calculations, the mutual inductance can be determined numerically by evaluating the change in flux linkage \( \Delta \Phi_m \) in a winding \( m \) in comparison to a reference value, as a result of a small change in current \( \Delta I_m \) in another winding \( n \)

\[
    M_{m,n} = \frac{\Delta \Phi_m}{\Delta I_m} \bigg|_{I_n=\text{const}}, \quad L_m = \frac{\Delta \Phi_m}{\Delta I_m}. \tag{13}
\]

The coupling coefficient is calculated as

\[
    k_{M(m,n)} = \frac{M_{m,n}}{\sqrt{L_m L_n}} \quad \tag{14}
\]

### II.VI. Predicting Coil Losses

Frequency-dependent coil losses can either be extracted from the real part of the coil voltage drop as predicted by the magnetic FEM simulation from the previous section, or be calculated analytically. Reatti and Bartoli et al. [17, 18] have developed an analytical expression for the AC resistance of solid and litz wire solenoids. For litz wire they arrive at
\[
R_{ac} = R_d \frac{\gamma_s}{2} \left\{ \begin{array}{l} 
\frac{1}{n_s} \left( \frac{\text{ber} \gamma_z \text{bei} \gamma_z - \text{be} i \gamma_z \text{ber} \gamma_z}{\text{ber}^2 \gamma_z + \text{bei}^2 \gamma_z} \right) \\
+ \left\{ (-2\pi) \left( 4 - \frac{N_s^2 - 1}{3} + 1 \right) n_s \left( \frac{\eta_1^2 + \eta_2^2}{2\pi n_s^2} \right) \right\} \right. \\
\left. \left( \frac{\text{ber} \gamma_z \text{bei} \gamma_z + \text{be} i \gamma_z \text{ber} \gamma_z}{\text{ber}^2 \gamma_z + \text{bei}^2 \gamma_z} \right) \right\}. 
\]
\]

where \( R_d = \frac{4N_i}{\eta_0 s d_c^2} \) is the DC resistance, \( N_i \) is the total number of turns, \( l_i \) the (average) length of a single turn, \( n_s \) is the strand count of the litz wire, \( d_c \) a single strand’s conductor diameter and \( \sigma \) its conductivity. \( \gamma_s = \frac{d_c}{s \sigma} \).

\( \delta^{-1} = \sqrt{\pi \mu_0 \sigma} \) is the skin depth, \( f \) the frequency and \( \mu_0 H_r \), the permeability of the strands. \( N_l \) is the layer count, \( \eta_1 = \frac{d_c v_z}{d_s} \) the external porosity factor, \( \eta_2 = \frac{d_c v_z}{d_i} \) the internal porosity factor, \( d_i \) the outer diameter of the litz wire bundle (without serving), \( t_o \) the distance between adjacent windings, \( t_s \) the spacing between strands, and \( p = N_l \frac{d_i}{d_s} \) is the litz wire packing factor. The porosity factors closely resemble filling factors. The porosity factor refers to the packing of strands in the litz bundle while the external porosity factor describes the arrangement of adjacent litz wires. \( \text{ber}_z, \text{bei}_z, \text{be} i_z \) are the Bessel-Kelvin functions of argument \( z \) and order \( v (v = 0 \) where omitted).

For the derivatives of the Kelvin functions, it is convenient to use the following identities [19]:

\[
\text{ber}_z' = -\text{ber}_{z+1} + \text{ber}_{z-1} - \text{be} i_{z+1} + \text{be} i_{z-1} \\
\text{bei}_z' = +\text{ber}_{z+1} - \text{ber}_{z-1} - \text{be} i_{z+1} + \text{be} i_{z-1} \\
\]

\[
\text{ber}_z, \text{bei}_z, \text{be} i_z \text{ are the Bessel-Kelvin functions of argument } z \text{ and order } v (v = 0 \text{ where omitted).}
\]

II.VII. Figures of Merit for Field Homogeneity

Salmon et al. [20] list several criteria for defining inhomogeneity. For MPI applications, the peak-to-peak field deviation \( IH_{pp} \) is the most critical value and should therefore be used for design evaluation:

\[
IH_{pp} = \frac{\max_{\Omega} (\vec{B}_r^\gamma \cdot \vec{\varepsilon}_z) - \min_{\Omega} (\vec{B}_r^\gamma \cdot \vec{\varepsilon}_z)}{|\vec{B}_r^\gamma \cdot \vec{\varepsilon}_z|} \\
\]

where \( \vec{\varepsilon}_z \) is the desired field direction and \( \vec{B}_r^\gamma \) the field at the center of the FOV. The vector \( \vec{\varepsilon} \) should be evaluated over the complete volume of interest (VOI).

In addition, we propose the directional inhomogeneity as another figure of merit that is useful for MPI applications. For systems with multiple drive field axes, the angle of the field of an axis should be evaluated in addition to its mere projection into the axes direction.

Again, we prefer to use a maximum deviation over average or root-mean-square (rms) values. We define the directional inhomogeneity \( IH_{DirMax} \) as

\[
IH_{DirMax} = \max_{\Omega} \left\{ \arccos \left[ \frac{\vec{B}_r^\gamma \cdot \vec{\varepsilon}_z}{|\vec{B}_r^\gamma \cdot \vec{\varepsilon}_z|} \right] \right\} \frac{180^\circ}{\pi}. 
\]

II.VIII. Equivalent Circuit Model and Network Solution

Figure 4 shows the equivalent circuit model we used to represent the distributed parasitics of the coil, which is loosely based on the work of Grandi et al. [6]. Its central building block are the inductances of the individual windings \( L_m \). To account for losses, we added the frequency-dependent resistors \( R_{ac} \) as well as regular resistors \( R_p \) parallel to the stray capacitances. Although it is reasonable to assume that these losses are frequency dependent as well in the case of a water cooled coil, there is no established model for these losses. While there are many works on the real part of the dielectric constant of water [21], there seems to be no equivalent for dielectric loss tangent in the kHz range. However, extending the model in such a way is easily accomplished, as the technique is the same as for \( R_{ac} \). The lossy capacitors are added between adjacent turns within the same layer \( C_{m,n} \) and between neighboring turns of adjacent layers \( C_{m,n+2} \). Since the coil in our case is not in proximity to a shielding structure, winding-to-shield capacitances are not included in Figure 4. For the FEM simulations, a shield can be easily modeled if required and incorporated into the model. Analytical calculation methods for wire-to-shield capacitances are given by Hoke et al. [7].
The (direct) stray capacitance between non-neighboring wires was neglected after FEM simulations predicted it consistently more than two orders of magnitude below the value of adjacent wires, owing to the distance as well as a shielding effect of the wire in between.

To accurately predict the coil impedance it is necessary to preserve the complete network structure, since higher resonances result from the complex interaction between stray capacitances and (mutual) inductances. Solving the network can either be performed by implementing a modified nodal approach [22], or by converting it into a net-list and using a SPICE [11] circuit solver. The latter approach has the additional benefit of directly implementing a simulation model of the coil that can be used in combination with other components, e.g. to simulate the behavior of complete filter circuits.

To account for the mutual inductances, a current controlled voltage source is added in series to the inducances of the windings. In the frequency domain, the induced voltage \( V_m \) in wire \( m \) as a result of the current changes in the other \( N \) windings \( n \) can be calculated from the branch voltages \( V_{l,n} \) across the corresponding inductors \( L_n \) through

\[
V_{m,m} = \sum_{n=1,\ n\neq m}^{N} M_{m,n} \frac{1}{L_n} V_{l,n}. \tag{20}
\]

However, if the SPICE program offers dedicated syntax to express coupling coefficients \( k_{M(m,n)} \), this approach is favored, to allow the circuit solver to optimize its matrix. Adding a voltage source for each mutual inductance (as opposed to adding a voltage source that is dependent on the derivatives of multiple node currents) should be avoided as it dramatically increases the node count of the circuit. Even though the resulting network equations will be the same (if the matrix is simplified), this will severely slow down the circuit analysis step that assembles the network matrices.

### III. Results

#### III.I. MPI Drive Coil Design

The previously described modeling methods and figures of merit were applied to optimize a MPI drive field generator for the use in a dual-frequency scanner in a series resonant circuit. A genetic algorithm [23, 24] was used to find a compromise between homogeneity, power losses, compact size and achievable field strength per unit current. An even layer count was favored to allow close proximity of the connecting wires.

As a result of the optimization, the cross section of the windings was arranged as a circle segment. To allow for a large radius of this circle and thus a compact design, additional windings (called reinforcement windings in Tab. 1) were added at either ends of the coil. The effect of these windings and the bend shape is that the homogeneity within the center region is better than for a straight solenoid coil of the same length. This allows easier access to the FOV (compared to longer coils) and reduces coil losses. Tab. 1 shows key data of the design.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total winding count</td>
<td>180</td>
</tr>
<tr>
<td>Reinforcement windings</td>
<td>5 on each end</td>
</tr>
<tr>
<td>Layers</td>
<td>3 (+1 reinforcement)</td>
</tr>
<tr>
<td>Inner coil diameter (min)</td>
<td>47.1 mm</td>
</tr>
<tr>
<td>Coil bend radius</td>
<td>875 mm</td>
</tr>
<tr>
<td>Coil length (winding area)</td>
<td>116.88 mm</td>
</tr>
<tr>
<td>Total wire diameter</td>
<td>2.03 mm</td>
</tr>
<tr>
<td>Wire serving</td>
<td>2×30 µm Mylar</td>
</tr>
<tr>
<td>Litz strands</td>
<td>420</td>
</tr>
<tr>
<td>Strand diameter</td>
<td>0.071 mm</td>
</tr>
<tr>
<td>Strand insulation</td>
<td>2.5 µm Polyurethane</td>
</tr>
<tr>
<td>Wire spacing</td>
<td>5 µm</td>
</tr>
<tr>
<td>( \epsilon_{in} = \epsilon_{Mylar} \approx \epsilon_{PUR} )</td>
<td>3.5 ( \epsilon_0 )</td>
</tr>
<tr>
<td>( \epsilon_{g} = \epsilon_{H,O} )</td>
<td>79.678 ( \epsilon_0 )</td>
</tr>
<tr>
<td>Field per unit current</td>
<td>1.667 mT A⁻¹</td>
</tr>
<tr>
<td>VOI for (18), (19)</td>
<td>20 mm × 20 mm</td>
</tr>
<tr>
<td>( H_{pp} )</td>
<td>0.676%</td>
</tr>
<tr>
<td>( H_{DirMax} )</td>
<td>0.235°</td>
</tr>
</tbody>
</table>

**Table 1:** Drive coil data

**Figure 5:** Drive coil design: CAD model with field overlay.

The coil was manufactured at the institute’s workshop. The coil former is made of Polyamide 6. To direct the flow of the coolant, the top winding layers were sealed using a thin layer of epoxy casting resin.

#### III.II. Impedance Without Coolant

Impedance measurements of the completed coil were carried out using an Agilent 4294A impedance analyzer with a 42941A probe. Fig. 6 shows a comparison between the measured impedance data \( Z_{Meas} \) and the impedance as predicted by the lumped component model with element values calculated using the FEM simulation \( Z_{FEM} \), as well as the analytical formulas \( Z_{analyt} \).

As is apparent from Fig. 6, the impedance of the coil is reasonably well predicted by the network up to the second series resonance at 3 MHz. Frequency-dependent
losses, however, are overestimated from that point onward so that the quality factor of the second series resonance ($Q_{\text{FEM,2nd,ser}} = 2.65$) is predicted lower than apparent from measured data ($Q_{\text{Meas.,2nd,ser}} = 8.56$). For frequencies below the second series resonance the ac losses show better agreement with the measured resistance, although losses are slightly overestimated there as well ($Q_{\text{FEM,2nd,par}} = 23.95$ vs. $Q_{\text{Meas.,2nd,par}} = 24.69$). The losses were determined by taking the real part of the impedance and by evaluating the quality factors of the resonances. The measurement uncertainty of the $Q$ factors should be below 4% for the settings used [25].

![Figure 6: Predicted impedance through analytical calculations ($Z_{\text{Analyt.}}$) and FEM simulations ($Z_{\text{FEM}}$) compared to the measured impedance ($Z_{\text{Meas.}}$) of the coil in air. An RLC model fit ($Z_{\text{RLC,fit}}$) to measured data is shown as well.](image)

Both analytical and FEM predictions show good agreement with measured data. To quantitatively compare the results with measured data, a RLC model (cf. Fig. 1) is fitted to both simulated and measured data and the resulting fit parameters are compared: The inductance value of the finite element simulation and the analytical formula show a $-0.8\%$ and $-0.2\%$ deviation from measured data respectively. Self-capacitance values show larger errors of $-7.9\%$ and $-11.0\%$ compared to measured results. As a consequence of the discrepancies, the self-resonance is shown $+4.6\%$ and $+5.8\%$ too high by the models.

![Figure 7: Predicted and measured impedance of the coil in water.](image)

Fig. 7 shows the impedance prediction compared to measured data for the water cooled coil. Again, the measured data agree with the predicted values by the finite element method up to the second series resonance. The first self-resonance shows very good agreement for the FEM data with $+1.22\%$ deviation, but larger errors for the analytical results (+10.9%). However, the frequencies of higher resonances are predicted 9% lower than actually measured. Another observation can be made from the quality factor of the higher resonances: Both analytical results and FEM values predict higher quality factors than those which are apparent from the measurement ($Q_{\text{FEM,2nd,par}} = 16.75$ vs. $Q_{\text{Meas.,2nd,par}} = 7.93$). This is in contrast to the slight over-prediction of losses that was found for the dry coil. A possible explanation for this behavior are losses from the coolant and additional proximity effect losses from displacement currents. The

III.III. Water-Cooled Coil

Water-cooling of coils fabricated from litz wire is often a compromise between low impact on the self resonant frequency and low thermal resistance between the windings and the coolant. To reduce the influence on stray capacitances, coated wires can be used. For the example shown here, the litz wire was covered by two layers of Mylar serving. The insulation thickness is assumed to be $65\,\mu m$, which is in agreement with a measured thickness of $30\,\mu m$ for a single Mylar foil layer. Experiments showed that the effect of the Mylar serving stems from its dielectric constant and not its tight sealing of the wires: Damaging the serving so that water can access the space between the strands (while keeping the serving mostly in place) only has minor influence on the self-capacitance of the coil.

$$Q_{\text{Meas.,2nd,par}} = 23.95 \quad \text{vs.} \quad Q_{\text{Meas.,2nd,par}} = 24.69.$$
deviations for the higher resonances may in part also be
due to the changed current paths in the coil. Since the
overall capacitances are much larger in the water cooled
case, displacement currents are increased in that case.

Fig. 2. However, the results of FEM simulations, as well as
the analytical formulas for the capacitance [7], (8) show
larger deviations (in the same direction) than the stag-
gered pattern. It was found that the FEM results in water
show acceptable agreement, but the self resonance for
the dry coil is predicted too high (cf. Fig. 9).

Figure 8: Analytical values compared to FEM reference results
for individual windings for (a) the coil in air, (b) the water-
cooled coil. The detail shows the upper end of the windings.
Numbers without units indicate the windings order.

Comparing analytical results and FEM simulation
data shows that the analytical formulas under-predict
the capacitance between wires. Fig. 8 shows the analyti-
cally calculated values and the differences compared to
FEM results for a detail of the coil. Values are positioned
between the windings to which they correspond. Devia-
tions are largest for windings at the edge of the winding
area. This observation might allow future improvements
in the analytical models to better account for media with
high dielectric constants.

III.IV. Stacked winding pattern

Based on the work of Hole et al. [7], we also performed
simulations where windings are stacked without stag-
gering, similar to the green marked winding pattern in

This can be explained through the fact that the stray
capacitances for the water-cooled coil are dominated
by the insulation and serving material’s permittivity and
thickness. Since the dielectric constant of water is very
high (ε_w = 79.678 ε_0) in comparison to that of most
insulators, this offsets the larger spacing in the stacked
configuration and the series connection of both capaci-
tances is still dominated by the insulation. This is true
in water for the chosen configuration up to a gap size
2S_g = 2.969 mm where both become equal with C_g =
C_m = 36.28 pF. The gap size plays a larger role in the
dry case with C_g being smaller than 36 pF for gap sizes
2S_g > 10.5 μm and thus dominating the series connec-
tion even with tight spacing. Therefore, large deviations
are observed in this case with the stacked pattern.

Apparently, the winding pattern of litz wire is bet-
ter described by the staggered arrangement. Most likely
this is in part due to the flexible nature of the used litz
wire, which will tend to form closer spacings than solid
wire would. Also, since the staggered pattern will always
appear as well for opposing winding pitches, the paral-
lel connection of both patterns will be governed by the
larger contribution, which will always correspond to the
staggered pattern.
IV. Conclusion

We have shown a practical method for predicting the impedance of liquid cooled multi-layered core-less solenoid coils by calculating the element values of a lumped component model and using the resulting network as a simulation model. Additionally, figures of merit have been identified for the water-cooled case, giving room to construct a water-cooled drive-field coil for a dual-frequency transmit chain, since parasitic resonances can limit the rejection bandwidth. This information is very beneficial when evaluating the frequency response of a complete MPI transmit chain, giving room for future refinements and research.

Acknowledgment

Financial support by the DFG under grant no. LU 800/5-1 and SCHI 383/2-1 is gratefully acknowledged.

References


