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Determination of 3D system matrices using a mirroring approach based on mixing theory

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Abstract

One approach of image reconstruction in MPI is the system matrix based reconstruction. With this approach, in addition to the particle behavior, the sequence and the scanner properties are also calibrated and stored in a system matrix, so that a linear system of equations for the image reconstruction must be solved. However, the measurement of the system matrix is very time-consuming, depending on the desired spatial resolution. Independently of this, there are some remarkable symmetries within the system matrix that could be exploited to significantly reduce the calibration time. In the context of this work the theoretical description of a system matrix about Chebyshev polynomials is used to completely build a 3D system matrix by mirroring an octant and to successfully reconstruct an image.

I Introduction

The state of the art for image reconstruction of MPI data acquired using a Lissajous based trajectory require a calibrated system matrix. This matrix contains all information about the used scanner, sequence and tracer material [1]. Depending on the number of measured spatial positions during the calibration, the acquisition is highly time consuming. For example, the acquisition of a system matrix on a 37 x 37 x 37 grid, which is part of the Open MPI dataset [2], took about 33h. The reduction of this time is highly relevant, since the acquisition of such a matrix is necessary for each nanoparticle type and each change during the scanning procedure. Furthermore, a scanner cannot be permanently blocked just for recording system matrices. One way to reduce the calibration time is to use the structural symmetries of the frequency patterns of a system matrix. In [3] a formula has been presented to mirror 2D system matrices and to reduce the number of needed calibration positions. In this work, we propose an alternative way based on the mixing order theory, which is also applicable for 3D system matrices.

II Material and methods

In previous works, the theory of the structure of a system matrix has already been applied. The theory is that the frequency components correspond to weighted Chebyshev polynomials of the 2nd kind, which encode the spatial distribution over a tensor product of the one-dimensional polynomials [1,4]. The Chebyshev polynomials are given as symmetric oscillating functions. These symmetries can also be observed in the frequency components as well. In Fig. 1 exemplary, a frequency component is shown to visualize the symmetries. Since a system matrix is complex, the data can be considered as magnitude and phase. In the shown layer horizontal and vertical symmetry is visible. It is particularly important to note that the correct determination of the center for the mirroring approach is crucial. This is particularly problematic for robot-based calibration, since everything must be accurate, whereas during field-based calibration [5] this problem can only occur to a limited extent.

Since each frequency component can be described by a theoretical mixture of 3 Chebyshev polynomials, they can also be used for symmetry. In general, each frequency component is given as

$$f_k = |m_x \cdot f_x + m_y \cdot f_y + m_z \cdot f_z|,$$

where $m_x, m_y, m_z \in \mathbb{Z}$ is the harmonic of the f_x, f_y, f_z frequencies. For the correct phase during mirroring, a factor $(-1)^{s_i}$ is multiplied globally on the mirrored part. Here, s_i, $i \in x, y, z$ is receive channel dependent comparable to the findings in [6]. Each s_i can be calculated by

$$\begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} = \begin{pmatrix} m_x+1 & m_y & m_z \\ m_x & m_y+1 & m_z \\ m_x & m_y & m_z+1 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix},$$

where $r_i = \begin{cases} 1, & \text{if } i \text{ is mirroring direction} \\ 0, & \text{else} \end{cases}, i \in x, y, z.$

Note, that the possible spatially dependent relaxation effects, which could additionally influence the phase, were neglected. For our experiments, we used the Open MPI dataset. We showed the applicability of the mirroring approach on a 37 x 37 x 37 system matrix and the reconstruction of a resolution phantom. Both are explicitly described in [2] and the datasets can be downloaded there as well. For the examination of the mirroring we used a single octant of the system matrix marked in Fig. 1 and proceeded according to the described instruction after the mirroring by a phase change. In the lower part of Fig. 1, the frequency component generated from one octant of the original system matrix is shown.

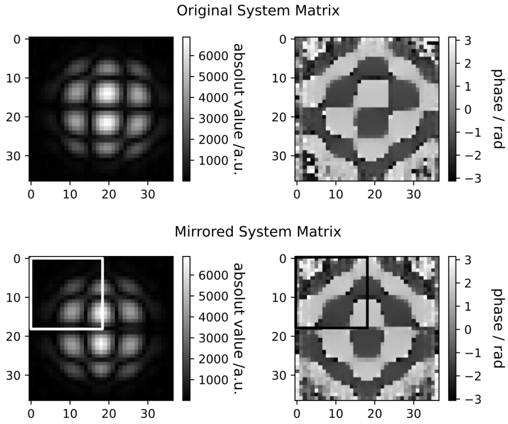

Figure 1: Comparison of the original (top) and of the mirrored (bottom) system matrix for one exemplary frequency component. Left the absolute value and right the phase of the frequency component are shown. Additionally, the top left marked corner in the mirrored system matrix represent the used octant for mirroring.

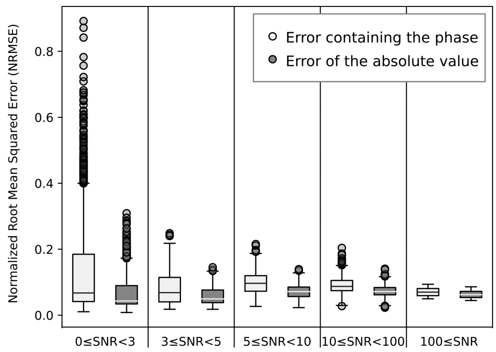

Figure 2: Boxplot of the NRMSE between the original and the mirrored system matrix sorted by SNR. On the left side, the calculated error contains the absolute value and the phase. On the right side, only the absolute values of each frequency components were compared.

III Results and discussion

To verify how similar the original and the mirrored system matrix are, we calculated the NRMSE for each frequency component. On the one hand, the absolute values only were compared and on the other hand, the complex frequency components were compared. The result is shown in Fig. 2 where we assigned the error to the SNR of the frequency components.

For orientation, the component shown in Fig. 1 had an SNR of about 24 and an NRMSE of 7 %. It should be noted that the error regarding the phase is greater than the absolute value only for all frequency components. The absolute value gives us an indication of the symmetry of the individual components, so that after subtracting the phase-including error from the absolute value, the frequencies are nearly phase accurate, which is necessary for a successful reconstruction. Furthermore, for small SNR values the error becomes very large, which is due to the fact that noise is not necessarily symmetric. As the SNR increases, the error also decreases.

To judge the impact on the image quality after reconstruction, we used the resolution phantom from the Open MPI datasets [2]. The reconstructed images are shown in Fig 3. For the reconstruction, only frequency components with an SNR greater than 3 were used, which is a common approach to neglect the noisy frequency components during reconstruction. Note that in Fig. 3 only one layer from the 3D dataset is shown. The reconstructed images are very similar, which is also confirmed by an NRMSE of the whole 3D volume of 3.6 %. Looking more closely at the images, small deviations are noticeable, which can also be traced back to the not yet ideal center finding and/or the neglected spatially dependent relaxation effects. A precise determination of the center

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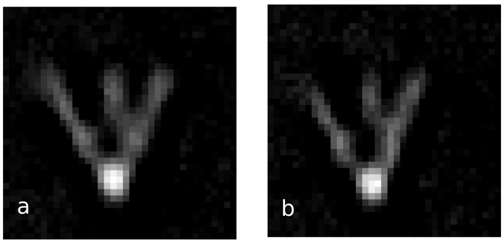

Figure 3: Reconstruction results using the original (a) and the mirrored system matrix (b).

is critical for the method, so that in the future even finer grids should be used and an accurate center determination must be performed.

IV Conclusions

In summary, a simple method was presented how 3D system matrices can be mirrored, saving up to 87.5 % of the calibration time. The time-savings can thus be used to measure a system matrix on a finer grid and/or with a higher averaging. In addition to mirroring within a system matrix, the approach can also be applied to multipatch data to reduce the number of system matrices to be explicitly acquired. Assuming that the field imperfections within the scanner are symmetrical, a system matrix with a corner in the scanner center would be sufficient to approximate the remaining seven system matrices.

Author's Statement

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