

Proceedings Article

# Parameter robustness analysis for system function reconstruction

A. Ö. Arol<sup>1,2,\*</sup> · A. A. Ozaslan<sup>1,2</sup> · A. Alpman<sup>1</sup> · E. U. Saritas<sup>1,2,3</sup>

<sup>1</sup>Department of Electrical and Electronics Engineering, Bilkent University, Ankara, Turkey

<sup>2</sup>National Magnetic Resonance Research Center (UMRAM), Bilkent University, Ankara, Turkey

<sup>3</sup>Neuroscience Program, Sabuncu Brain Research Center, Bilkent University, Ankara, Turkey

\*Corresponding author, email: [arol@ee.bilkent.edu.tr](mailto:arol@ee.bilkent.edu.tr)

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## Abstract

The quality of images in system function reconstruction (SFR) depends on an extensive calibration scan that acquires the system matrix (SM). A change in parameters of the scanner, trajectory, or magnetic nanoparticle requires the acquisition of a new SM. In this work, we analyze the parameter robustness of SFR with simulations. We investigate the effect of utilizing an existing SM in the case of a change in the aforementioned parameters. The results show that a new calibration scan is not needed, as long as the change is sufficiently small.

## I Introduction

System function reconstruction (SFR) is one of the main image reconstruction methods for magnetic particle imaging (MPI) [1-2]. In SFR, the system matrix (SM) characterizes the mapping between the received MPI signal and magnetic nanoparticle (MNP) distribution at each position in the field-of-view (FOV). However, SM is specific to scanner and trajectory parameters, as well as the physical parameters of MNP. A change in any one of these parameters requires a new calibration scan, which can be time consuming.

In this work, we analyze the parameter robustness of SFR. With simulations, we investigate the effects of using an existing SM (i.e., without a new calibration) in the case of a change in one of the abovementioned parameters. Specifically, we look at how a change in the trajectory parameters such as FOV size and trajectory density, or a change in MNP size affects the performance of SFR. The aim of this analysis is to determine the extent of change in parameters that SFR can withstand without a new calibration.

## II Material and methods

We performed simulations in MATLAB to investigate the effects of using mismatched parameters during calibration and imaging. First, we simulated calibration scans on a 40x40 grid for an MPI system with 2cmx2cm FOV. Selection field gradients were (3,3,-6) T/m/ $\mu_0$  along (x,y,z) direction. Single-core monodisperse MNP with 25-nm diameter was assumed. A Lissajous trajectory in x-y plane was created using the following 2D drive field (DF):

$$\begin{aligned} H_x(t) &= H_p \sin(2\pi f_0 t) \\ H_y(t) &= H_p \sin(2\pi f_1 t) \end{aligned} \quad (1)$$

Here,  $f_0 = 25$  kHz and  $f_1 = 24.75$  kHz was utilized, with  $H_p = 30$  mT/ $\mu_0$  to cover the desired FOV.

SFR seeks the solution to an ill-posed inverse problem, requiring regularization for stable reconstruction. In our reconstructions, we considered the regularized weighted least-squares problem, employed Tikhonov regularization, and used weightings according to the row energies of SM [3]. Accordingly, the rows of SM with energies less than 4 % of the energy of the row with max-

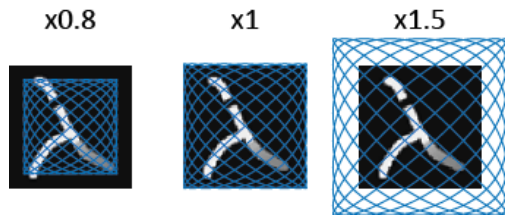


Figure 1: Effect of changing DF amplitude on the covered FOV.

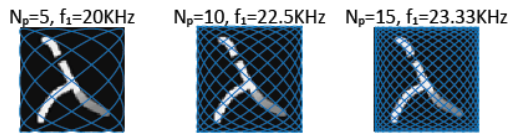


Figure 2: Sample trajectories with small  $N_p$  values for illustration purposes. Much denser trajectories were used in actual simulations.

imum energy were removed. In addition, frequencies up to  $1.5 \cdot f_0$  (i.e., fundamental frequency bands) were also filtered out. The reconstructed image expressed as the solution to the following optimization problem:

$$\operatorname{argmin}_x \|W^{\frac{1}{2}}Ax - W^{\frac{1}{2}}b\|_2^2 + \lambda \|x\|_2^2, \quad (2)$$

where  $A$  is the SM,  $b$  is the received signal,  $\lambda$  is the Tikhonov regularization parameter, and  $W$  is a diagonal matrix with entries equals to squared reciprocals of the row energies. Here, Eq. 2 was solved using the Kaczmarz method, and  $\lambda$  was chosen for each case separately.

After obtaining the SM and reconstruction results for the abovementioned parameters, the following changes were considered to evaluate the parameter robustness of SFR:

### II.I Change in MNP Diameter

Ideal MNP magnetization curve is modeled by the Langevin function [4], which is a strong function of the MNP diameter. In day-to-day usage of an MPI scanner, there may be a need to change the MNP used, e.g., due to availability of a better MNP or for testing the performance of a new MNP. Here, the MNPs were assumed to be single-core and monodisperse, with diameter varying between 10-35 nm. Relaxation effects were ignored.

### II.II Change in FOV Size

During imaging, one may want to change the FOV size to cover a wider region than initially foreseen, or to zoom into a region of interest. Here, FOV size was varied by changing the DF amplitude  $H_p$ . As exemplified in Fig. 1, the FOV covered varied between 40 % and 200 % of the original FOV for which the SM was acquired.

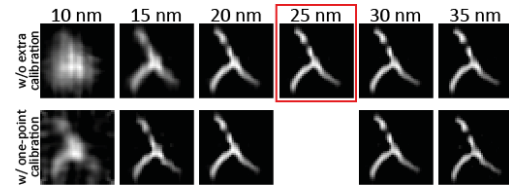


Figure 3: [Top] Reconstructed images for different sizes of particles using an SM acquired for 25 nm. [Bottom] Improvements achieved with a one-point correction.

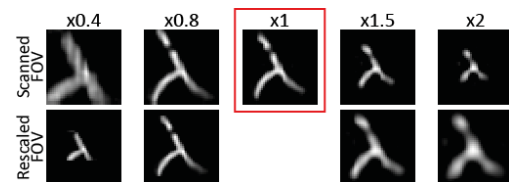


Figure 4: [Top] Reconstructed images for different FOV sizes. [Bottom] Same images rescaled to the correct FOV size.

## II.III Change in Trajectory Density

Frequency ratio of the two DFs in Eq. 1 determines the trajectory density,  $N_p$ , as follows:

$$f_0 = \frac{N_p}{(N_p - 1)} f_1. \quad (3)$$

As exemplified in Fig. 2, depending on the ratios of DF frequencies, both the duration of the trajectory ( $T_R = N_p / f_0$ ) and the distances between sampled points change. Hence,  $N_p$  affects the reconstructed image considerably. Previous work on trajectory analysis has shown that higher  $N_p$  yields higher resolution MPI images for both SFR and x-space reconstructions [5-6]. Therefore, during imaging, one may want to increase  $N_p$  to achieve a higher quality image. Here,  $N_p$  was varied while keeping  $f_0$  fixed. Note that this change requires the frequency of the second coil (i.e.,  $f_1$ ) to change. However, the change in frequency is relatively small and can still fall within the resonant band of that coil.

## III Results and Discussion

The results of using an existing SM in the case of a parameter change are shown below.

### III.I Effect of Changing MNP Diameter

Figure 3 shows the results of utilizing a MNP with a diameter different than the one in the calibration scan. When a larger MNP is utilized, the performance of SFR does not change visibly, whereas using a smaller MNP reduces the image quality considerably. This result implies that reconstructions from a better MNP are restricted by the



**Figure 5:** Resulting images for different  $N_p$  values.

quality of the existing SM, whereas reconstructions from a lower quality MNP are restricted by the performance of the MNP itself.

A change in MNP diameter changes the effective size of the field free point, as well as the ratios between the signals at higher harmonics. Here, we tested the effectiveness of a simple, one-point correction to compensate for the change in higher harmonic ratios. Accordingly, a point source containing the new MNP was placed at the origin, and a calibration scan was acquired *for this point only*. The existing SM at all other positions were then corrected to match the higher harmonic ratio of this one-point calibration. As shown in Fig. 3, a considerable improvement in image quality is achieved for 10-nm and 15-nm diameter cases. A similar analysis and correction may be utilized when incorporating nanoparticle relaxation effects.

### III.II Effect of Changing FOV Size

As shown in Figure 4, no severe artifacts were seen in the reconstructed images for different FOV sizes. However, for bigger FOVs, the effective size of a pixel gets larger, causing a loss of resolution in the image. This work considered ideal magnetic fields and homogenous coil sensitivities. A similar analysis can be performed to observe the effects of non-ideal magnetic fields.

### III.III Effect of Changing Trajectory Density

Changing  $N_p$  changes the relative timing of the trajectory, where the same point in the FOV is traversed at a different time point. As shown in Fig. 5, a modest 5 %-10 % change

in  $N_p$  does not cause any severe artifacts. In contrast, larger changes in  $N_p$  causes a warping/remapping effect in the reconstructed image, as different portions of the MNP distribution is mapped to incorrect locations.

## IV Discussions and Conclusions

In this work, we analyzed the effects of scanning parameter changes on the reconstructed images for SFR. The results show that small changes on scanning and nanoparticle parameters do not require new calibration scans, as they yield comparable image quality. We have also shown that the quality of reconstructions can be improved by using a one-point calibration scan.

## Author's Statement

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