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Bias-reduction for sparsity promoting regularization in magnetic particle imaging

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Abstract

Abstract: Magnetic Particle Imaging (MPI) is a tracer based medical imaging modality with great potential due to its high sensitivity, high spatial and temporal resolution, and ability to quantify the tracer concentration. Image reconstruction in MPI is an ill-posed problem that can be addressed by regularization methods that each lead to a bias. Reconstruction bias in MPI is most apparent in a mismatch between true and reconstructed tracer distribution. This is expressed globally in the spatial support of the distribution and locally in its intensity values. In this work, MPI reconstruction bias and its impact are investigated and a two-step debiasing method with significant bias reduction capabilities is introduced.

I Introduction

MPI is a quantitative medical imaging method which is based on the non-linear magnetization response of magnetic nanoparticles to static and dynamic magnetic fields [1]. Image reconstruction in MPI can be formulated as an ill-posed inverse problem, which requires regularization methods to be solved [2]. Tikhonov is the most commonly used regularization functional in MPI. Other typical regularization functional choices in imaging applications are L^1 and total variation (TV) [3]. The former is usually used to promote sparsity and the latter is applied to preserve edges. However, the use of regularization introduces bias in the reconstructed signals and images and has been addressed for some medical imaging techniques, such as, PET/CT [4]. This work aims to investigate bias in MPI and introduce a method to reduce it. Therefore, we adopt a two-step debiasing method [5].

II Material and methods

The first step of the two-step debiasing method proposes to obtain the support of the true solution by solving the standard variational problem

$$c_\alpha = \operatorname*{argmin}_c \frac{1}{2}||Sc-u||_2^2 + \alpha J(c), \tag{1}$$

with a convex regularization functional J and a regularization parameter $\alpha > 0$. While the second step aims to reduce the bias by minimizing the data fidelity term over all the c values that share the support of the first step solution. This sharing support condition can be reformulated using a vanishing Bregman distance constraint

$$\hat{c}_\alpha \operatorname*{argmin}_c \frac{1}{2}||Sc-u||_2^2 \quad \text{s.t} \quad D_J^{p_\alpha}(c, c_\alpha), \tag{2}$$

where $P_\alpha \in \delta J(c_\alpha)$. The first-order primal dual algorithm proposed in [6] is used to solve the two minimization problems (1) and (2).

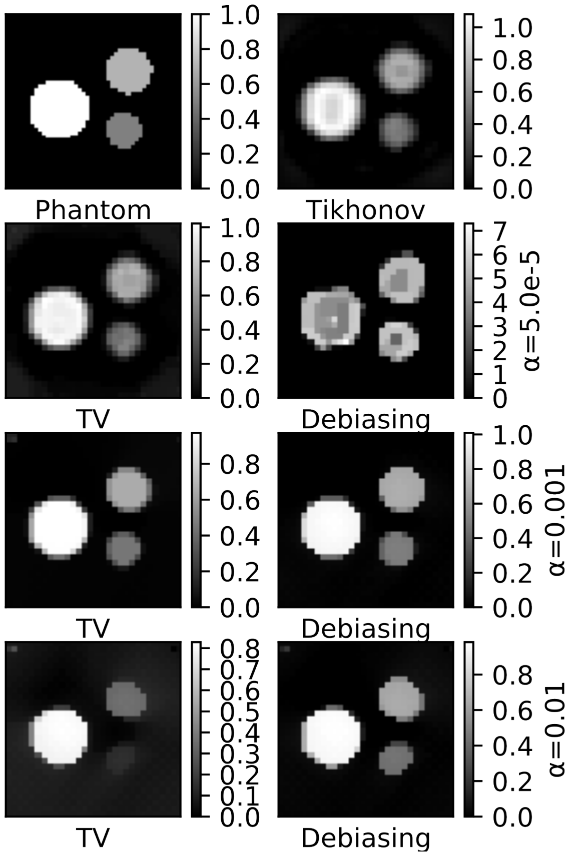

Figure 1: TV reconstruction vs. Bregman debiasing.

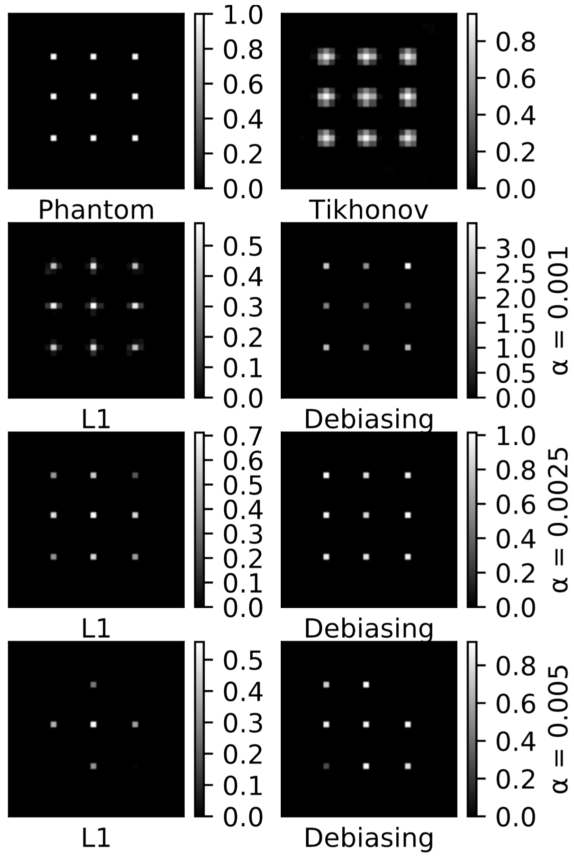

Figure 2: L^1 reconstruction vs. Bregman debiasing.

The bias and its reduction are investigated using a three circles phantom for TV regularization and a sparse dots phantom for L^1 regularization in a simulation study. The phantoms are shown in Fig. 1 and Fig. 2 for TV and L^1 regularization, respectively.

III Results

III.I Analysis of reconstruction bias

The first row of Fig. 1 and Fig. 2 display the original phantoms and their Tikhonov reconstruction using 10 Kaczmarz iterations. We notice that the converged Kaczmarz solution of the circles phantom has a non-homogeneous structure and suffers from an overshooting in the intensities and the Kaczmarz solution of the dots phantom is blurred and has a drop in the intensities.

The other three rows of the first column of Fig. 1 and Fig. 2 display the TV and L^1 reconstruction results for three different regularization parameter values, respectively. Fig. 1 shows that the TV reconstruction result for $\alpha = 5 \cdot 10^{-5}$ successfully recovers the correct intensities of the original phantom but fails to get a homogeneous, edge preserving solution. For $\alpha = 0.001$, we notice that the phantom structure is better recovered but its values are biased, i.e. we obverse an overall drop in the intensities. Increasing α to 0.01 causes over-regularization and a further reduction of the signal intensities. For $\alpha = 0.001$ in Fig. 2, the L^1 reconstruction is able to recover the sparse structure of the phantom but significantly overestimates the intensities. The choice of $\alpha = 0.0025$ results in a better recovery of the overall phantom and lower deviation in the intensity values compared to the other reconstructions shown. When α value is increased further to 0.005, the over-regularization effect causes some of the dots of the phantom to disappear and an underestimation of the intensities.

III.II Reduction of reconstruction bias

The second columns of Fig. 1 and Fig. 2 show the debiased results for the circles phantom and the dots phantom, respectively. Starting with Fig. 1, for $\alpha = 5 \cdot 10^{-5}$, the debiasing step has an overall negative impact on the reconstruction due to the noise amplification and the overestimation of intensities. While the debiasing step for $\alpha = 0.001$ results an optimal solution with nearly perfect intensities recovery and an overall improvement in the phantoms structure.

For $\alpha = 0.01$, the debiased solution yields better intensities but still shows over-regularization. From Fig. 2, the

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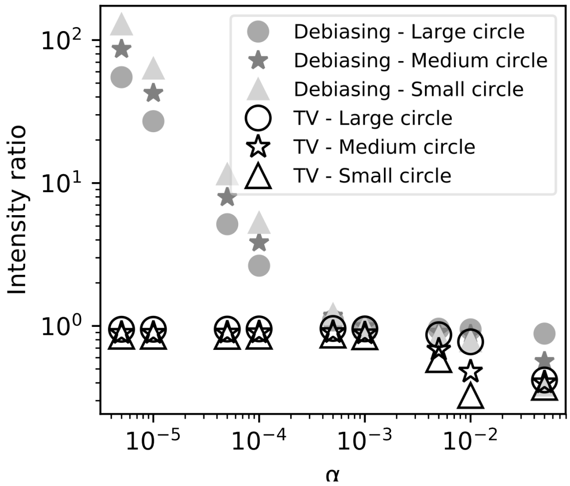

Figure 3: The sum of the intensities for each circle relative to the value obtained from the original circles phantom for different α values.

debiasing step yields the optimal solution for $\alpha = 0.0025$, a nearly perfect recovery of the intensities values and a perfect recovery of the phantoms structure. However, for $\alpha = 0.001$ the solution intensities are overestimated and some noise is reintroduced in the debiased result and for $\alpha = 0.005$, the solution is improved but still affected by the over-regularization of the L^1 result. As a measure for the bias reduction success, in Fig. 3 we divided the sum of the intensities for each circle of the circles phantom by the sum of the intensities of the large, medium, and small circles in the original phantom which are 216, 90.3, 40.5, respectively. This implies that the closer the intensity ratio is to 1, the less biased is the reconstruction. For the optimal regularization parameter, $\alpha = 0.001$, the intensities are 206, 80.1, 34.0 before and 214, 89.5, 39.7 after debiasing. A similar improvement can be observed for all α values greater than the optimal one, where the summed intensities are closer to the ones of the phantom after debiasing. For small α values, where the impact of the regularization term is neglectible, the ration is close to one. Here, debiasing leads to a significant overestimation of the summed intensities.

IV Discussion and Conclusion

For small α values the debiasing step should not be used as it increases the bias compared to the non-debiased reconstruction. For sufficiently large α values the debiasing step actually reduces the bias but stays affected by the over-regularization in the first step since it operates on the support of the non-debiased reconstruction. Therefore, with an appropriate choice of the regularization parameter the proposed two-step debiasing method yields an effective reduction of bias and thus an improvement in the quantifiability of the reconstruction.

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