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Enhanced compressed sensing recovery of multi-patch system matrices in MPI

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Abstract

In magnetic particle imaging, many applications require the time consuming measurement of a system matrix before image reconstruction. Reduction of measurement time can be achieved with the help of compressed sensing, which is based on the sparsity of the system matrix in a suitable transform domain. In this work, we propose regularization functions to exploit the additional correlations in multi-patch system matrices. Experiments show that the resulting recovery method allows successful matrix recovery at higher undersampling factors than a standard compressed sensing recovery.

I Introduction

Magnetic particle imaging (MPI) is a radiation-free imaging method, which allows imaging the distribution of magnetic nanoparticles at both high spatial and high temporal resolution [1]. Most acquisition schemes in MPI require the measurement of a system matrix before image reconstruction can take place. For 3d imaging experiments such measurements can take multiple days, which is impractical for many applications. Moreover, the scanner needs to be kept stable during the measurement. Depending on the scanner architecture this poses a challenge yet to be solved.

The scan time for this calibration can be significantly reduced by using compressed sensing techniques (CS) [2,3,4]. The latter exploit the fact that the rows of the system matrix become sparse after applying transformations such as a fast Fourier transform or a discrete cosine transform (DCT). In combination with an incoherent sampling pattern, CS allows to recover a complete system matrix from a highly undersampled measurement.

In this work, we investigate the use of CS for the recovery of multi-patch system matrices. The latter play an important role in MPI as they allow increasing the field of view (FOV) without incurring peripheral nerve stimulation. Based on the similarity of multi-patch system matrices, we propose regularization functions to exploit inter-patch correlations. Our results show that the proposed algorithm allows successful recovery at higher undersampling factors, when compared to existing single-patch CS methods.

II Material and methods

II.I Properties of multi-patch system matrices

In the following, let $\hat{\mathbf{s}}_k \in \mathbb{C}^{n_v \times n_p}$ denote the k^{th} frequency component of a multi-patch system matrix. Here, n_v denotes the number of voxels measured on a d-dimensional grid and n_p denotes the number of patches. One then finds that for all patches, the patterns in $\hat{\mathbf{s}}_k$ have a very similar structure. In Fig. 1, this can be seen for four exemplary patches of a system matrix measured with the MPI brain imager presented in [5]. When ap-

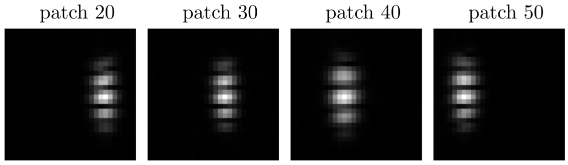

Figure 1: Multiple patches of the 8th frequency component of the measured system matrix. Shown are the patches 20, 30, 40 and 50.

plying a DCT to the patterns, one observes that the DCT coefficients have similar values at the same respective position. In order to recover signals with that characteristic, joint-sparsity regularization based on the $l_{2,1}$-norm

$$||\alpha||_{2,1} = \sum_{i=1}^{n_v}\left(\sum_{p=1}^{n_p}\alpha_{i,p}^2\right)^{\frac{1}{2}} \qquad (1)$$

is a common tool.

Moreover, we observe that $\hat{\mathbf{s}}_k$ can be represented as a tensor of order $(d+1)$ with low multi-rank. Here, the multi-rank refers to the size of the core-tensor in the higher-order singular value decomposition (HOSVD) of $\hat{\mathbf{s}}_k$. This is a generalization of the results presented in [6]. To recover signals with such a structure, one can use a generalized nuclear norm $||\hat{\mathbf{s}}_k||_*$, which corresponds to the l_1-norm applied to the elements of the core tensor of the HOSVD.

II.II Matrix recovery

Based on aforementioned observations, we formulate the following problem for the recovery of multi-patch system matrices

$$\text{argmin}_{\hat{\mathbf{s}}_k} ||y_k - P\hat{\mathbf{s}}_k||_2^2 + \lambda_{sp}||\Phi\hat{\mathbf{s}}_k||_{2,1} + \lambda_{lr}||\hat{\mathbf{s}}_k||_*. \qquad (2)$$

Here y_k contains the measured points for the k^{th} frequency component and P denotes the corresponding sampling pattern. Φ is a block-diagonal operator, applying a type-II DCT to each patch. For solving problem (2) we use the Split-Bregman method [7].

II.III Validation on a measured system matrix

To validate the proposed method, we use a system matrix measured on a 28x28 grid with a FOV of 140x140 mm_2 and 130 patches. The system matrix was measured with a 5x5x10 mm_3 sized delta sample filled with Perimag. The system matrix was retrospectively undersampled using a Poisson disk pattern with undersampling factors ranging from two to five. Fully sampled system matrices were reconstructed using problem (2). For comparison, we computed solutions for the $l_{2,1}$- regularized case ($\lambda_{lr} = 0$) and

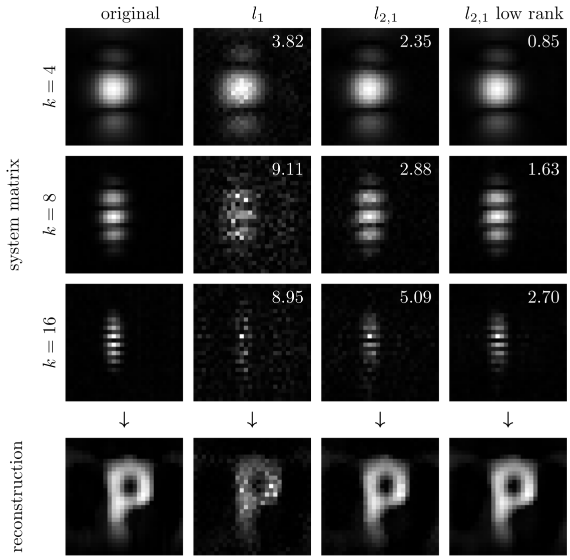

Figure 2: Recovered patterns of patch 40 for three representative frequency components (top). The superimposed numbers denote the NRMSD of the recovered pattern with respect to the fully sampled reference (in %). The bottom shows the images reconstructed with the corresponding system matrix.

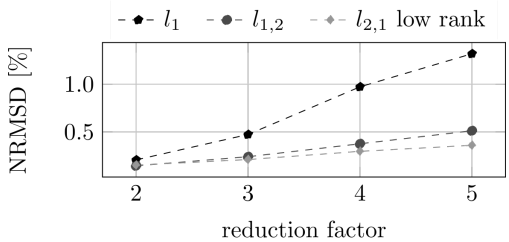

Figure 3: NRMSD of recovered system matrices in dependence of the undersampling factor. The NRMSD was calculated with respect to the fully sampled system matrix.

for the case that both regularization parameters are nonzero. As a reference, we also performed a standard CS recovery with an l_1-penalty of the DCT coefficients. For a quantitative comparison of the results, we calculated the normalized root mean squared deviation (NRMSD) of each recovered pattern with respect to the fully sampled reference. In each case, the regularization parameters we chosen manually, such that the mean NRMSD of the recovered frequency components was minimized.

As a further test, the recovered system matrices were used to reconstruct images from a measurement of a P-shaped phantom filled with Perimag. Reconstruction was performed using the l_2-regularized Kaczmarz algorithm with 200 iterations and a relative regularization parameter of 0.5.

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III Results and discussion

Recovered patterns for some representative frequency components, and an undersampling factor of five, are shown in Fig. 2. As can be seen, the l_1-based CS method is able to recover the 4^{th} frequency component, albeit with some residual noise. The other frequency components are severely corrupted by artifacts. In contrast, the proposed methods successfully recover all frequency components. While the reconstruction using only the $l_{2,1}$-term displays some residual noise for the 16^{th} frequency component, even this noise is removed when using the additional higher order low rank regularization. Those observations are also reflected in the values for the NRMSD, which are overlayed in Fig 2.

Additionally, Fig. 3 shows the NRMSD of the recovered system matrices in dependence of the undersampling factor. As can be seen, the error rapidly increases for the l_1-based method, while the other methods exhibit a much slower increase. Hence, one can conclude that the use of inter-patch correlations allows for a successful recovery of system matrices at higher undersampling factors.

For the image reconstruction experiment, we obtained a very noisy image when using the system matrix recovered with the l_1-based CS method. In contrast, both proposed methods yield high quality reconstructions.

Finally, it should be noted that inter-patch correlations could also be exploited in different ways. One example is the design of a suitable $(d+1)$-dimensional sparsifying transform. However, such a transformation requires further information on the acquisition scheme. Hence, such an approach would probably be less generic than the proposed method.

IV Conclusions

We developed a method for the joint recovery of multi-patch system matrices from severely undersampled measurement. The proposed method exploits additional correlations in a multi-patch system matrix and allows for successful recovery at larger undersampling factors.

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