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A sparse row-action algorithm for magnetic particle imaging

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Abstract

The image reconstruction in Magnetic Particle Imaging (MPI) relies on efficiently solving an ill-posed inverse problem. Current state-of-the-art reconstruction methods are either based on row-action methods with fast convergence but limited noise suppression or advanced sparsity constraints showing better image quality, but suffering from a higher computational complexity and slower convergence. In this contribution, we propose a novel row-action framework where advanced sparsity constraints, e.g., a combination of l_1- and TV-norm, can be included. Its performance is numerically evaluated on simulated and real MPI data, showing a significant reduction of computation time while retaining the enhanced imaging quality.

I Introduction

In Magnetic Particle Imaging (MPI), the reconstruction of the tracer concentration is commonly based on a system matrix approach. With the measured system matrix $A \in \mathbb{C}^{m\times n}$, the measurement vector $b \in \mathbb{C}^m$ the reconstruction of the tracer concentration $x \in \mathbb{C}^n$ amounts to solving

$$Ax = b\,.$$

This ill-posed problem is usually solved using a regularized Kaczmarz approach based on Tikhonov regularization. In terms of MPI, this method exhibits a fast convergence due to near orthogonality of the system matrix. On the downside, however, Tikhonov regularization results in smoothed edges, limited noise suppression as well as reduced contrast [1]. In order to overcome these issues, Storath et al. proposed the non-negative Fused Lasso by introducing a combination of l_1- (Lasso) and total variation (TV) regularization priors [1]. The improved image quality that results from the total variation approach comes with a higher computational complexity compared to the classical Kaczmarz approach. This is quantified by a factor of seven in [1]. In this contribution, we introduce a general sparse row-action framework for inconsistent linear systems, where the combination of l_1/TV priors can be included into the regularization problem. Instead of the usual soft-thresholding operator that results from the l_1-norm penalty, we suggest to use the non-negative Garrote (NNG), which is shown to significantly improve the reconstruction results. Numerical evaluation is based on simulated MPI data and experimental data from the OpenMPIData initiative [2].

II Material and methods

Let $f(x) = ||Ax - b||_2^2$ and let R be a lower semicontinuous and convex penalty function, e.g., we consider a combination of l_1- and TV norm in the following. Minimizing $f(x) + R(x)$ with respect to x can be done using the forward-backward splitting, given by the following iteration $x_{k+1} = prox_{\gamma R}(x_k - \gamma \Delta f(x_k))$ for some $\gamma > 0$. This scheme is characterized by its gradient (forward)

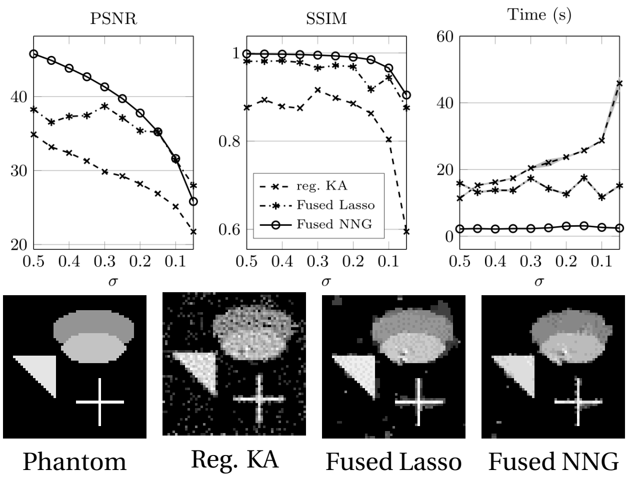

Figure 1: Top: PSNR, SSIM and computation time for the regularized Kaczmarz, the Fused Lasso and the Fused NNG approach in dependence of the noise level σ. Bottom: Reconstructed tracer concentration for $\sigma = 0.05$.

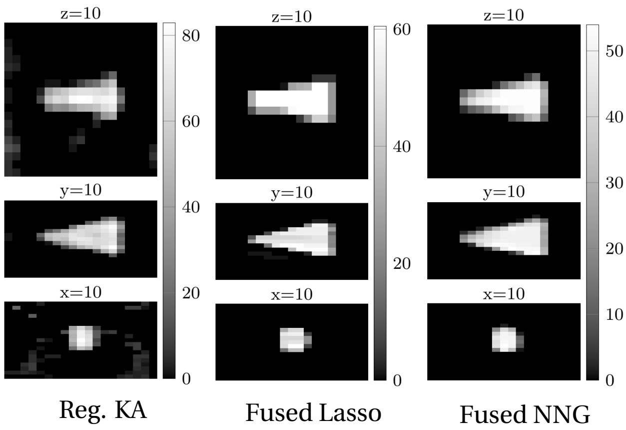

Figure 2: 3D reconstruction results in mmol/l.

step using f and its backward step based on the proximity operator of R. If $R = 0$ the forward-backward splitting with constant $\gamma = ||A||_2^{-2}$ results in the Landweber iteration. With $a_i^T \in \mathbb{C}^n$ denoting the i-th row of system matrix A, this motivates the following Kaczmarz based iteration

$$y_{k+1} = x_k + \frac{b_i - \langle a_i, x_k \rangle}{||a_i||_2^2} a_i^* \tag{1}$$
$$x_{k+1} = \begin{cases} prox_R(y_{k+1}) & \text{if } \operatorname{mod}(k,m) = 0 \\ y_{k+1} & \text{if } \operatorname{mod}(k,m) \neq 0 \end{cases}$$

where $i = \operatorname{mod}(k,m)$. This iteration consists of repeatedly doing a Kaczmarz step sweeping over all rows of the system matrix, followed by a proximal step. In order to solve the minimization problem

$$\min_{x \geq 0} \frac{1}{2} ||Ax - b||_2^2 + \lambda_1 ||x||_1 + \lambda_2 ||x||_{TV}, \tag{2}$$

the proximity operator of the corresponding penalty term $R = \lambda_1 ||x||_1 + \lambda_2 ||x||_{TV}$ is given by the well-known proximity operator of the isotropic TV-norm (the 2D/3D algorithm introduced in [3] is used in the subsequent experiments) followed by a soft-thresholding (see [7] for a proof of this splitting in a multidimensional setting). Substituting the soft-thresholding with the Nonnegative Garrote (NNG) thresholding [4] defined by

$$N_\lambda(x) = x \cdot \max\left(1 - \frac{\lambda^2}{|x|^2}, 0\right)$$

leads to our proposed Fused NNG algorithm. This changes the above minimization problem, however, the corresponding penalty term for the NNG thresholding operator still leads to sparse solutions [4]. Note that the basic Kaczmarz iteration in our proposed algorithm (1) only works for consistent linear systems [5]. In case of inconsistent systems, such as in MPI, the Kaczmarz iteration has been shown to converge whenever the noise of the inconsistent system is quite low [Thm. 3.2, 6]. In the case of MPI, SNR thresholding of the system matrix can achieve such a low noise situation [5].

The experimental results in the following section are based on a measured 2D Magnetic Particle Spectrometry (MPS) system matrix (12mT in x- and y-direction, discretized to a grid size of 57x57). Forward simulations for a phantom x (Fig. 1, left) are given by $b = A(\sigma x) + \eta$, with weights $\sigma \in (0,1)$ and a noise vector η which consists of averaged empty scanner measurements. The smaller σ, the larger the influence of the background η. From the resulting b only frequencies with SNR larger than 1.5 and within the frequency range of 70-300 kHz are considered for reconstruction. The resulting system matrix has size 3065x3249. Comparison algorithms are the regularized Kaczmarz (reg. KA) [5] and the Fused Lasso [1]. Although all three algorithms do not solve the same minimization problem, they are included in order to put the performance of our Fused NNG into perspective with state-of-the-art MPI reconstruction algorithms. The stopping criterion of all algorithms is defined to be the relative change between two iterations, i.e., $||x_k - x_{k+1}||_2 ||x_k||_2^{-1} < 3 \cdot 10^{-4}$. Quality measures are the PSNR, SSIM (optimal SSIM value is one) and an averaged computation time (over 50 runs). Parameters λ_1 and λ_2 for all algorithms are chosen to maximize the PSNR.

III Results and discussion

At top of Figure 1, the resulting quality measures for the simulated data set in dependence on the noise level σ are shown. Our proposed approach gives the best recon-

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struction results, except for the largest noise level in the PSNR plot. The computation time for our Fused NNG is approximately two seconds for all noise level.

Fig. 1 (bottom) illustrates the reconstructed tracer concen-tration for the largest noise level $\sigma = 0.05$. It shows the limited noise suppression of the regularized Kaczmarz and that the Fused Lasso approach still has some artefacts in the background (black parts of the phantom) of the image, and explains the poorer SSIM values compared to our approach. The reconstructed values of the triangle (original value 0.0375) have a mean of 0.0365±0.0078 for the reg. KA, 0.0370±0.0014 for the Fused Lasso and 0.0356±0.0029 for our Fused NNG. This translates to the slightly better PSNR of the Fused Lasso for this noise level.

Results on experimental 3D MPI Data are based on the data sets provided by the OpenMPIData initiative [2]. The shape phantom is used, which is a 3D cone filled with perimag (50 mmol/l). For reconstruction, only frequencies larger than 70 kHz and with a SNR larger than four are considered. This results in a system matrix of size 3870x6859. The reconstructed cone shape is visualized in Fig. 2 in layer view. The homogeneity of the tracer concentration inside the cone favors the Fused Lasso in the xy-plane and our approach in the xz- as well as yz-plane. In terms of absolute reconstructed tracer concentration, both approaches lead to similar results close to the original tracer concentration of 50 mmol/l, with the Fused Lasso having slightly larger outliers.

The computation time for the regularized Kaczmarz is 37.5 seconds and 27.3 seconds for the Fused Lasso. Our proposed Fused NNG, on the other hand, cuts the computation time down to approximately 2.4 seconds.

IV Conclusions

In summary, we have shown that the proposed sparse row-action approach outperforms current state-of-the-art MPI reconstruction algorithms in terms of PSNR and SSIM. The Fused NNG approach reduces computation time significantly, providing more accurate reconstruction results that are better suited in real-time applications such as catheter navigation in interventional radiology. The extension of the NNG-thresholding to account for neighboring pixel information is one of the next targets.

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