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Boundary artifact reduction by extrapolating system matrices outside the field-of-view in joint multi-patch MPI

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Abstract

In multi-patch magnetic particle imaging an artifact-free image can be obtained by using a joint reconstruction and measuring the system matrices not only in the field-of-view but also in a huge overscan. This leads to a long calibration time and heavy memory consumption and therefore an unsuitability of this method for large three-dimensional measurements. In this work, we propose to only measure the system matrices in the field-of-view and to use a diffusion based extrapolation step to extend the system matrices computationally into the overscan. In this way, we massively reduce the calibration time while maintaining a nearly artifact-free image.

I. Introduction

For measuring the spatially dependent particle response with magnetic particle imaging (MPI), a field-free point (FFP) is moved along a certain trajectory through the region of interest, defining the so called field-of-view (FOV) [1]. The size of the FOV is limited due to physiological constraints and too small for human applications. To overcome this limitation, a static multi-patch concept is used by adding a homogeneous focus field onto the gradient field, measuring a volume via the drive field and then shifting the FOV in space to the next position acquiring the next patch. Excitation of particles outside the FOV is one of the major obstacles in this approach. Using a joint reconstruction algorithm, it is possible to gain an artifact free reconstruction for non-overlapping patches [2, 3]. However, for this approach the system matrices must not only be acquired within the FOV of each patch but need to cover a certain area around each FOV, named overscan, in which the signal smears out [2, 4]. Especially for large 3D measurements this is very time- and memory consuming. In this work, we propose a method where the system matrices need to be only acquired within the FOV. In a post-acquisition step the system matrices are extrapolated into a large overscan by a diffusion-ansatz with homogeneous Dirichlet boundary conditions to reduce the boundary artifacts.

II. Material and methods

In the following, we consider a two-dimensional multi-patch configuration with two non-overlapping horizontally aligned quadratic patches as illustrated on the left side in Figure 1. We denote the joint area of FOV and overscan as Ω^i for patch $i \in \{1,2\}$.
The goal is to use a simple diffusion method to extrapolate the system matrices without any overscan. Such an extrapolation is in particular feasable, if the signal in the overscan is only a harmonic continuation of the signal in-

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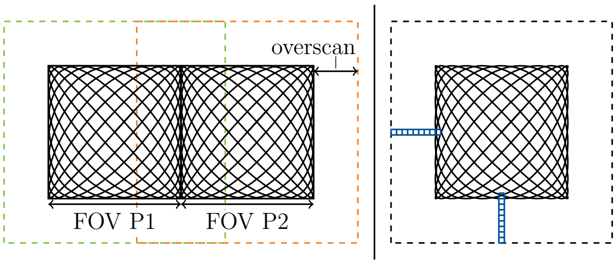

Figure 1: Left: schematic representation of a two-dimensional multi-patch configuration with two non-overlapping patches; the overscan of the patches is shown by the dashed lines. Right: blue lines denote the pixels from the FOV into the overscan on which the smearing of signal is measured by calculating the effective rank.

side the FOV and will vanish evantually. To quantify and investigate the approach, the concept of effective dimensionality is used, measured by the effective rank erank(·) [5]. Let the frequency vector $\boldsymbol{S}_l^i$ be the l-th column in the system matrix of patch i for $l \in \{1,\ldots,L\}$, $L \in \mathbb{N}$. We study straight spatial lines of pixels from the border of the FOV into the overscan in x- and y-direction as shown on the right side in Figure 1 and collect the corresponding frequency vectors $\boldsymbol{S}_l^i$ as columns into the matrices $\boldsymbol{S}_{\boldsymbol{x}}^i$ and $\boldsymbol{S}_{\boldsymbol{y}}^i$. An effective rank of one would imply a complete linear dependence and therefore indicates that the signal propagation from the FOV into the overscan can be well emulated by an harmonic continuation where the signal fades out. On the other hand, an effective rank near to the normal rank of the matrix would imply a strong linear independence between the frequency vectors.

We consider system matrices for each frequency component $k \in \{1,\ldots,K\}$, $K \in \mathbb{N}$ truncated to the FOV (denoted by $\tilde{\boldsymbol{S}}_k^i$ for patch $i \in \{1,2\}$) and the cost function

$$J\left(\boldsymbol{V}_k^i\right) := \left\|\nabla \boldsymbol{V}_k^i\right\|_2^2, \tag{1}$$

which has to be minimized for $i \in \{1,2\}$ over $\boldsymbol{V}_k^i$, $\forall k \in \{1,\ldots,K\}$ using homogeneous Dirichlet boundary conditions. We propose two variants of the minimization problem. Variant I is the basic extrapolation:

$$J\left(\boldsymbol{V}_k^i\right) \xrightarrow{\boldsymbol{V}_k^i} \min, \tag{2}$$

under the constraints

$$\boldsymbol{V}_k^i\big|_{\mathrm{FOV}} = \tilde{\boldsymbol{S}}_k^i \quad \text{and} \quad \boldsymbol{V}_k^i\big|_{\partial\Omega^i} = 0. \tag{3}$$

Variant II keeps few single measured values in both x- and y-direction as additional constraints to improve the extrapolation while still reducing the data samples in the overscan severely.

III. Experiments

The proposed method was tested on two-dimensional system matrices, which were recorded with the preclinical MPI-scanner of Bruker with a gradient strength of $-1\,\mathrm{T\,m^{-1}}$ in x- and y-direction, $2\,\mathrm{T\,m^{-1}}$ in z-direction and drive field amplitudes of 12 mT in x- and y-direction leading to a composed FOV of $24 \times 48\,\mathrm{mm}^2$. A point sample of size $1\,\mathrm{mm}^3$ filled with perimag with an iron concentration of 10 mg/ml was used for the aquisition of 40×40 gridpoints per patch, using an overscan of 8 mm in each direction. The system matrices were considered at all frequencies higher than 60 kHz with an SNR above 5, truncated to the FOV and extrapolated by variant I and II. For variant II, a total of 16 measured values in the overscan were kept per patch: two lines with two data points placed equidistantly along each direction.

The method was also evaluated on the basis of reconstructed images. To this end, we measured a bar of size $30 \times 1 \times 5\,\mathrm{mm}^3$ filled with perimag with an iron concentration of 10 mg/ml, placed diagonally over the patch intersection. The reconstruction was solved by a joint multi-patch approach [3] with 8 Kaczmarz iterations and a relative regularization parameter of $\lambda = 0.01$.

IV. Results and discussion

The calculation of the effective rank on the measured data for $\boldsymbol{S}_{\boldsymbol{x}}^{\boldsymbol{i}}$ and $\boldsymbol{S}_{\boldsymbol{y}}^{\boldsymbol{i}}$, $i \in \{1,2\}$ collected over nine pixels gave

$$\begin{aligned} \operatorname{erank}(\boldsymbol{S}_{\boldsymbol{x}}^{\boldsymbol{1}}) &= 2.93, \quad & \operatorname{erank}(\boldsymbol{S}_{\boldsymbol{y}}^{\boldsymbol{1}}) &= 2.69, \\ \operatorname{erank}(\boldsymbol{S}_{\boldsymbol{x}}^{\boldsymbol{2}}) &= 2.78, \quad & \operatorname{erank}(\boldsymbol{S}_{\boldsymbol{y}}^{\boldsymbol{2}}) &= 2.79, \end{aligned}$$

which is for both patches and in both directions much smaller than the normal rank of nine and promised good results for the extrapolation step.

For a qualitative analysis of the method on the system matrix level we exemplarly consider the frequency component $k = 69$, because it has a lot of signal in the overscan. In Figure 2 it can be seen that both variants are able to continue the wave pattern into the overscan correctly. Variant II is closer to the original, as espected. For a short quantitative analysis we consider the root mean squared error (RMSE) of the system matrices compared to the original one over the whole frequency range:

$$\mathrm{RMSE}_{\mathrm{tr}} = 0.89, \quad \mathrm{RMSE}_{\mathrm{vI}} = 0.52, \quad \mathrm{RMSE}_{\mathrm{vII}} = 0.42,$$

which is an improvement of 41.1 % for variant I and 52.4 % for variant II.

By looking on the reconstructed images in Figure 3, the artifacts on the patch boundaries generated by truncating the system matrices to the FOV can be clearly seen in the upper right image. The presented extrapolation method is able to strongly reduce the artifacts in both variants.

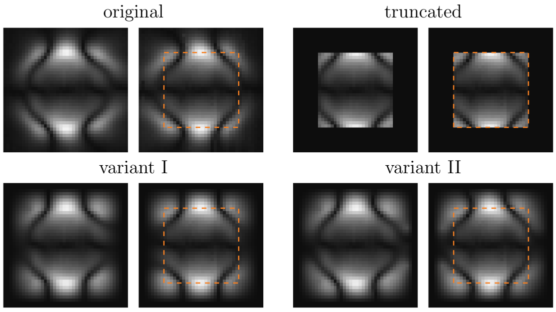

Figure 2: Comparison of the absolute values of the system matrices of both patches for frequency component $k = 69$. Patch 1 is on the left side, patch 2 on the right side. The dashed line denotes the boundary between FOV and overscan.

Especially variant II gives a very good reconstruction result. For a quantitative error analysis the RMSE of the different reconstructed images compared to the original one is considered. For the RMSE we get

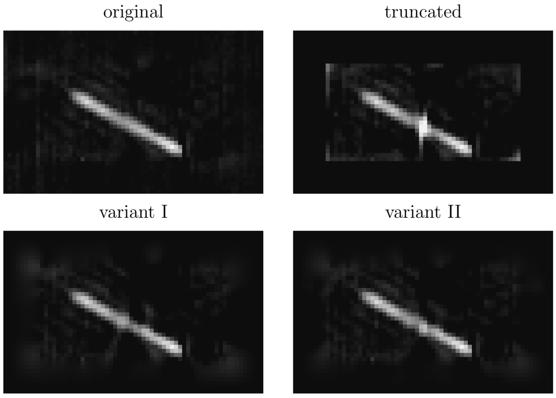

Figure 3: Reconstructed multi-patch images with the original, the truncated and in both variants extrapolated system matrices using a joint algorithm.

$$\mathrm{RMSE}_{\mathrm{tr}} = 0.037, \quad \mathrm{RMSE}_{\mathrm{vI}} = 0.019, \quad \mathrm{RMSE}_{\mathrm{vII}} = 0.014,$$

which is an improvement of 48.7 % for variant I and 61.6 % for variant II. In the case of artifact investigation it is also meaningful to consider the maximum norm error (MaAE) of the reconstructed images compared to the original one, since strong artifacts may be mistaken as particle signal. A good reduction of the maximum of the pixelwise absolute difference to the original reconstructed image is therefore crucial for the performance of the extrapolation method. We get

$$\mathrm{MaAE}_{\mathrm{tr}} = 28.92, \quad \mathrm{MaAE}_{\mathrm{vI}} = 9.66, \quad \mathrm{MaAE}_{\mathrm{vII}} = 8.17,$$

which is an improvement of 66.6 % for variant I and 71.8 % for variant II. This supports the foregoing considerations that both proposed variants of the extrapolation method can achieve good reconstruction results. Especially variant II enables nearly artifact free reconstruction by reducing the system matrix acquisition points by 63.25 %. The system matrix acquisition of the overscan took about 20 minutes per patch. In comparison, the calculation step takes only a few miliseconds leading to a strong reduction of the total calibration time and could be performed directly before the reconstruction. This leads to a possible memory reduction proportional to the reduction of acquisition points.

V. Conclusions

The proposed extrapolation step enables nearly artifact-free reconstruction by strongly reducing the time usage for system matrix acquisition. For the considered 2D multi-patch experiment we achieved good results with 63.25 % less acquisition points compared to the reconstruction with a large measured overscan. The extrapolation step is also applicable in three dimensions and enables artifact-free reconstruction for large multi-patch 3D measurements by reducing the calibration time and memory consumption drastically. For further reduction the proposed method can be combined with system matrix warping [6] on multi-patch MPI in future work.

Author's statement

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References

[1] T. Knopp, N. Gdaniec, and M. Möddel. Magnetic particle imaging: From proof of principle to preclinical applications. *Physics in Medicine and Biology*, 62(14):R124, 2017.

[2] T. Knopp, K. Them, M. Kaul, and N. Gdaniec. Joint reconstruction of non-overlapping magnetic particle imaging focus-field data. *Physics in Medicine and Biology*, 60:L15, 2015, doi:10.1088/0031-9155/60/8/L15.

[3] P. Szwargulski, M. Möddel, N. Gdaniec, and T. Knopp. Efficient Joint Image Reconstruction of Multi-Patch Data reusing a Single System Matrix in Magnetic Particle Imaging. *IEEE Transactions on Medical Imaging*, 38(4):932–944, 2019.

[4] A. Weber, F. Werner, J. Weizenecker, T. M. Buzug, and T. Knopp. Artifact free reconstruction with the system matrix approach by overscanning the field-free-point trajectory in magnetic particle imaging. *Physics in Medicine and Biology*, 61(2):475–487, 2015, doi:10.1088/0031-9155/61/2/475.

[5] O. Roy and M. Vetterli. The effective rank: A measure of effective dimensionality. *European Signal Processing Conference*, pp. 606–610, 2007.

[6] M. Boberg, T. Knopp, and M. Möddel. Reducing displacement artifacts by warping system matrices in efficient joint multi-patch magnetic particle imaging. *International Journal on Magnetic Particle Imaging*, 6(2 Suppl 1), 2020, doi:10.18416/IJMPI.2020.2009030.

10.18416/ijmpi.2022.2203019