International Journal on Magnetic Particle Imaging
Vol 8, No 1, Suppl 1, Article ID 2203032, 4 Pages

Infinite Science Publishing

Proceedings Article

Averaging Randomized Kaczmarz for Magnetic Particle Imaging

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Abstract

Magnetic particle imaging (MPI) is a promising modality for medical imaging with high resolution and sensitivity. In this paper, the averaging randomized Kaczmarz (ARK) reconstruction method is proposed to accelerate the image reconstruction process. Compared with the traditional Kaczmarz method, the ARK algorithm improves the image quality and reconstruction resolution. The iterative parameters of the ARK are selected, and further optimization needs to be studied.

I. Introduction

Magnetic particle imaging (MPI) is a promising medical modality that images the concentration distribution of superparamagnetic iron oxide (SPIO) nanoparticles in vivo with high resolution, sensitivity, and safety [1]. An efficient MPI reconstruction algorithm is of great importance to ensure the reconstruction accuracy and quality, and many studies have been conducted to optimize the algorithm [2–4]. There are two main reconstruction methods: the system matrix (SM) method and the x-space method [5, 6], and the SM method is more accurate [7].

The SM is regarded as the discretized MPI system function, which describes the complex and detailed relationships between the input concentration distribution of the SPIO and the output voltage signal. The Kaczmarz method is a widely used iterative solver in the SM-based MPI image reconstruction, and it operates on a single row of the matrix at a time and then sweep all rows [5]. Using the rows of the matrix in Kaczmarz's method in random order can greatly improve the rate of convergence than in the given order [8]. However, this strategy uses sequential updates, making parallel computation difficult. So optimized iterative solvers are needed to speed up the convergence.

In this paper, we introduce an averaging randomized Kaczmarz (ARK) method for MPI reconstruction, which can improve the rate of convergence and get high-quality reconstruction images. We begin by showing the details of the ARK algorithm and then compare its reconstruction results with that of the Kaczmarz method. Some problems and prospects are also mentioned. And finally, we summarize this work.

II. Methods and materials

II.I. Kaczmarz method in MPI

In the SM method, the key reconstruction problem becomes to solve an inverse and ill-posed matrix equation, which is expressed as:

$$SC = U \tag{1}$$

with system matrix $S := (s_{m,n}) \in \mathbb{C}^{M\times N}$, the desired SPIO concentration distribution $C := (c_n) \in \mathbb{R}_+^N$, and the measured voltage signal in frequency domain $U := (u_m) \in \mathbb{C}^M$.

To solve the SM equation, the Kaczmarz iterative algorithm is used. Its sub-iteration can be formulated as

$$c^{k+1} = c^k + \frac{u_j - s_j^* c^k}{\|s_j\|_2^2} s_j^* \tag{2}$$

where k is the sub-iteration and u_j is the j-th entry of U. There are different ways to select the row index j, and in the Kaczmarz method, the simplest way is used by increasing j after every sub-iteration and then sweeping through all the matrix rows in cyclic manner [9]. However, this selection way has slower convergence rate, and it can be accelerated by sweeping the rows in random order, which is called randomized Kaczmarz (RK) [8].

II.II. Averaging Randomized Kaczmarz

The standard RK method is difficult to do parallel computation and lead low rate of convergence[8]. To break this limitation, the ARK method uses a weighted average of independent updates to parallel computing. At each iteration, an equation is picked out at random from the system matrix in (1) and this iterate is projected onto the solution space of that equation. And a weighted average of the updates is defined as the iterative step, and then the step is taken in the direction of this projection. Specially, we write the ARK iteration formula:

$$c^{k+1} = c^k + \frac{1}{q}\sum_{j\in\tau_k} \omega_j \frac{u_j - s_j^* c^k}{\|s_j\|_2^2} s_j^* \tag{3}$$

where τ_k is a random combination of q row indices and $\omega_j = 1$ represents the weight of the j-th row, uniformly distributed. Note that, the rows are rearranged according to $\|s_j\|_2^2/\|s\|_2^2$, and then q rows are selected and randomly ordered into τ_k. The ARK algorithm is explained in detail in the following:

1. Input $S := (s_{m,n}) \in \mathbb{C}^{M\times N}$, $U := (u_m) \in \mathbb{C}^M$, $c^0 \in \mathbb{C}^N$, $\omega \in \mathbb{C}^M$, the maximum iteration number K, selected row numbers q
2. for $k = 0, \dots, K-1$ do
3. $\tau_k \leftarrow q$ indices sampled from $\|s_j\|_2^2/\|s\|_2^2$
4. Compute $\delta_k \leftarrow \frac{1}{q}\sum_{j\in\tau_k} \omega_j \frac{u_j - s_j^* c^k}{\|s_j\|_2^2} s_j^*$ in parallel
5. Update $c^{k+1} \leftarrow c^k + \delta_k$
6. Output c^K

III. Experiments

In this study, the MPI data we used comes from the open MPI datasets, University Medical Center Hamburg-Eppendorf, Hamburg, Germany [10]. We use the 3D MPI data of the resolution phantom, which consists of five tubes. The initial calibration scan is measured at $37\times37\times37$ grid positions, and for better visualization, the reconstruction results are interpolated as $128\times128\times128$ grid positions.

To select the appropriate parameters of the ARK, the Peak Signal to Noise Ratio (PSNR) and the Root Mean Square Error (RMSE) are calculated under different parameter values. The first group of experiments is to change the iteration number in the range of 10 to 3000, keeping the number of row indices q constant, and then the Kaczmarz method and the ARK method are used to reconstruct. The second group is to change the number of row indices q from 1 to 200 and calculate the results of the Kaczmarz and the ARK with the same iterations. After that, using the optimized parameters, the three-dimensional reconstruction is conducted by the ARK method and the Kaczmarz method, and the images are shown in Section IV.

IV. Results

For both the Kaczmarz and the ARK method, the PSNR and the RMSE curves of different iteration numbers are shown in Figure 1. Keep $q = 200$. With increasing the iteration number, the PSNR of the ARK is always higher than that of the Kaczmarz and the RMSE of the ARK is lower than that of the Kaczmarz, which means the ARK converges fast with a more accurate solution.

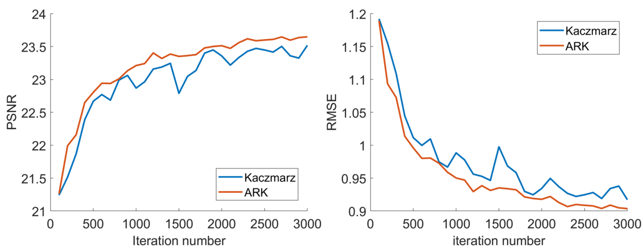

Figure 1: (left) PSNR curve of different iteration number; (right) RMSE curve of different iteration number

Figure 2 shows the PSNR and the RMSE curves of different q in the ARK method. Keep the iteration number equal to 2000 to receive a better convergence solution.

The PSNR reaches its maximum point and the RMSE meets its minimum value under the same row indices number, where q equals 21. That is to say, selecting 21 rows to form a random set can make the ARK method in the appropriate iteration step.

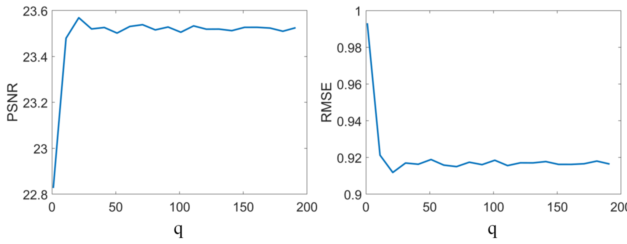

Figure 2: (left) PSNR curve of different q; (right) RMSE curve of different q

The image reconstruction results of the ARK method and the Kaczmarz method are shown in Figure 3, with $K = 2000, q = 21$. In the ARK results, the straight tubes of the resolution phantom can be identified, while those of the Kaczmarz are deformed. So the ARK method improves the reconstruction spatial resolution with rapid convergence.

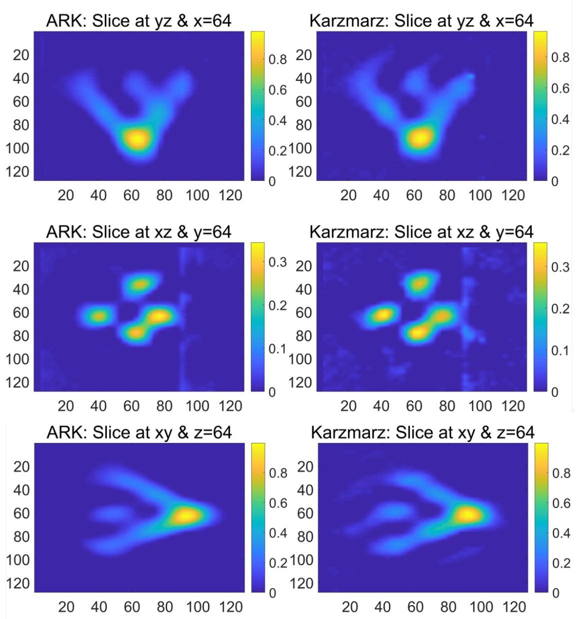

Figure 3: Reconstruction images:(left) ARK method; (right) Kaczmarz method

V. Discussion

In this study, we assume the weights ω and the probabilities $\|s_j\|_2^2/\|s\|_2^2$ are independent of each other to simplify the iteration producer. However, these two components are both contribute to the iteration error calculation. Since we want the final concentration error of the ARK to converge to zero, there will be a relationship between the weights and the probabilities, limiting the error to zero. Then the coupling of weights and probabilities need to be further studied.

VI. Conclusion

To satisfy the quality requirements of the MPI reconstruction, the averaging randomized Kaczmarz method is proposed to speed up the convergence and improve the accuracy of the solution. We select the optimized iteration parameters of the ARK algorithm and reconstruct the Open MPI data with higher resolution. The ARK has been preliminarily proved to be effective in system matrix reconstruction of MPI, and further improvements needs to be considered.

Acknowledgments

This work was supported in part by the National Key Research and Development Program of China under Grant 2016YFC0103803, 2017YFA0700401, and 2017YFA0205200; Natural Science Foundation of China under Grant: 62027901; CAS Youth Innovation Promotion Association under Grant 2018167, and CAS Key Technology Talent Program.

Author’s statement

Authors state no conflict of interest. Informed consent has been obtained from all individuals included in this study. Conflict of interest: Authors state no conflict of interest. Informed consent: Not applicable. Ethical approval: Not applicable.

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10.18416/ijmpi.2022.2203032