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Image Reconstruction in Magnetic Particle Imaging Based on Gaussian Weighted Laplace Prior Regularization

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Abstract

Magnetic particle imaging (MPI) is an emerging tomographic imaging modality with high spatial and temporal resolution. The image reconstruction in MPI needs to solve an ill-posed inverse problem. Tikhonov regularization is known to help solve this kind of problems. However, the traditional Tikhonov regularization (L2 regularization) guides the reconstruction to be over smooth and tends to generate a lot of low-value scattered signals around the real reconstructed objects. In this work, we develop an efficient and noise reducing reconstruction method for MPI. We propose a Gaussian weighted Laplace regularization which assumes that the correlation between any two voxels inside the field of view (FOV) has a non-linear inverse relationship with their spatial distance. Experimental results show that the proposed method can provide a more accurate MPI reconstruction.

I. Introduction

As an emerging imaging modality, magnetic particle imaging (MPI) utilizes the non-linear magnetization response of magnetic nanoparticles (MNPs) to provide the distribution of MNPs [1]. In contrast to other tomographic methods, MPI does not employ any ionizing radiation and offers a high spatial and temporal resolution.

In MPI, the reconstruction of the MNPs distribution from the measured voltage signal is an ill-posed inverse problem [2]. Tikhonov regularization is known to help solve this problem [2, 3]. The traditional Tikhonov regularization (L2 regularization) guides the reconstruction to be over smooth and tends to generate a lot of scattered signals around the real reconstructed objects [4].

The reconstruction performance can be improved by improving the Tikhonov regularization. In [2], the authors introduced a diagonal weighting matrix in the Tikhonov regularization to weight the system matrix and the measurement vector to improve the reconstruction performance. In this study, we propose to add a Gaussian weighted Laplace regularization prior to the regularization matrix in the Tikhonov regularization and solve the optimization problem based on the Kaczmarz [2] method (GL-KZ). Experimental results show that GL-KZ method can provide more accurate MPI reconstruction.

II. Methods and materials

II.I. The linear system of MPI

In MPI, the measured induced voltage in the receiving coil reflects the reaction of MNPs in the magnetic field-free region. The voltage is related to the total magnetization **M** of MNPs and the receive coil sensitivity **P**:

$$u(t) = -u_0 \int_{\mathbf{V}} \mathbf{P}(\mathbf{r}) \cdot \frac{\partial \mathbf{M}(\mathbf{r,t})}{\partial t} d^3 r \qquad (1)$$

where u(t) represents the received voltage time domain signal. **V**, **r** and t denote the FOV volume, the position vector and the time, respectively. For MPI reconstructions, the voltage signal u(t) is transformed to frequency domain. A linear mathematical model can be used to describe the relationship between the measured voltage signal and the distribution of MNPs:

$$\mathbf{Ax} = \mathbf{U} \qquad (2)$$

where $\mathbf{A} \in \mathbb{C}^{M \times N}$ denotes the system matrix measured in advance, which reflects the relationship between the measured voltage and the real MNPs distribution [1, 2, 5]. $\mathbf{x} \in \mathbb{C}^{N \times 1}$ is the distribution vector of MNPs. $\mathbf{U} \in \mathbb{C}^{M \times 1}$ is the Fourier transform of the measured voltage. N is the number of the pixels in the FOV. M is the number of the used frequency components.

II.II. Gaussian weighted Laplace prior regularization

The Tikhonov regularization is frequently used to help solve the inverse problem Eq. 2 and complete the image reconstruction. By integrating different regularization matrices, Tikhonov regularization can effectively stabilize the inverse problem and restrict a solution. The optimization problem is as follows:

$$\hat{\mathbf{x}} = \arg\min_{\mathrm{x}} \ ||\mathbf{Ax}\text{-}\mathbf{U}||_2^2 + \lambda ||\mathbf{Lx}||_2^2 \qquad (3)$$

where $\hat{\mathbf{x}}$ denotes the optimal solution. λ and **L** represent the regularization parameter and regularization matrix, respectively. The most commonly used regularization matrix **L** is the identity matrix and this regularization method is known as the L2 regularization. However, the L2 regularization has a trend of smoothing, which further makes the reconstructed region larger than the real region and generates many scattered signals. Therefore, to overcome this problem, the spatial neighborhood structure is to be taken into account as a prior to construct the regularization matrix.

In this study, we take the neighborhood structure into consideration as a prior, which implies that neighboring voxels' intensities are assumed to be correlated. In particular, this correlation is inversely proportional to the spatial distance. Figure 1 further describes this spatial structure. Take the voxel V_0 in space as an example, the correlation between it and neighboring voxels decreases as the spatial distance increases.

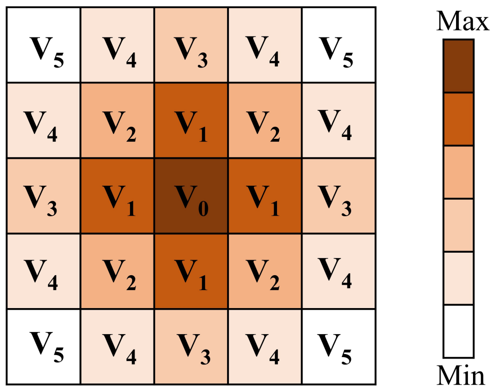

Figure 1: A sketch of the neighborhood structure shows an inverse relationship between the center voxel V_0 and other voxels.

Based on the neighborhood structural prior, we construct a regularization matrix. The regularization matrix L is defined as $\mathbf{L} = (l_{i,j})_{N \times N}$:

$$l_{i,j} = \begin{cases} 1, & i = j \\ -\exp(-\frac{d_{i,j}^2}{4R^2}), & j \in O(i), i \neq j \\ 0, & \text{otherwise} \end{cases} \qquad (4)$$

where $O(i)$ denotes the neighborhood of voxel i, and Euclidean distance between voxel i and j is denoted by $d_{i,j}$. Furthermore, the derivation of R is detailed in [6]. In this study, we defined the neighborhood according to the spatial distance. As shown in Figure 1, for any voxel V_0 in imaging space, its neighborhood contains the voxels of $V_0 - V_5$. The iterative Kaczmarz method is used [2] to solve the inverse problem.

III. Experiments

To evaluate the performance of the proposed method, we compare the reconstruction results of L2 regularization and our proposed Gaussian weighted Laplace regularization based on the OpenMPI data set (#6) [7]. The FOV has a size of 37mm×37mm×18.5mm and sampling in x-, y- and z-direction, resulting in 37×37×37=50653 voxels, which equals to N, the number of columns in the system matrix A. Besides, we only consider the frequencies which are larger than 80KHz and SNR larger than 4. Hence, the resulting system matrix A has a size of 2118×50653.

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In order to qualitatively evaluate the reconstruction results, we construct true data based on the phantom data provided in the OpenMPI data set. To enhance the accuracy of the true data, the size of the ground truth is 128×128×128 and the reconstructed results are also interpolated as 128×128×128 based on the bilinear interpolation method. Mean squared error (MSE) and structural similarity (SSIM) are used as the evaluation metric.

IV. Results

We solve the inverse problem Eq. 3 based on the Kaczmarz method [2]. For Kaczmarz method, the regularization parameter λ and number of iterations have a huge influence on the performance. As reference in [2], Kaczmarz method gets good results with just a few iterations and the regularization parameter can be chosen based on the results after convergence. Therefore, we set the regularization parameter and number of iterations as 0.1 and 20 in this study, based on which the results are the bset.

The reconstructed results of the two methods are shown in Figure 2. It can be seen that the results of L2 regularization method show the scattered state and a lot of low-value scattered signals appeared in the background, which can be clearly observed from the 3D reconstruction results. In contrast, our proposed method accurately reconstructs the phantom and the background is very clean. We also compare quantitative indicators: MSE: GL-KZ vs L2-KZ = 4.7 vs 5.04; SSIM: GL-KZ vs L2-KZ = 0.949 vs 0.932. The proposed method has obvious advantages both in terms of demonstration results and quantitative analysis results.

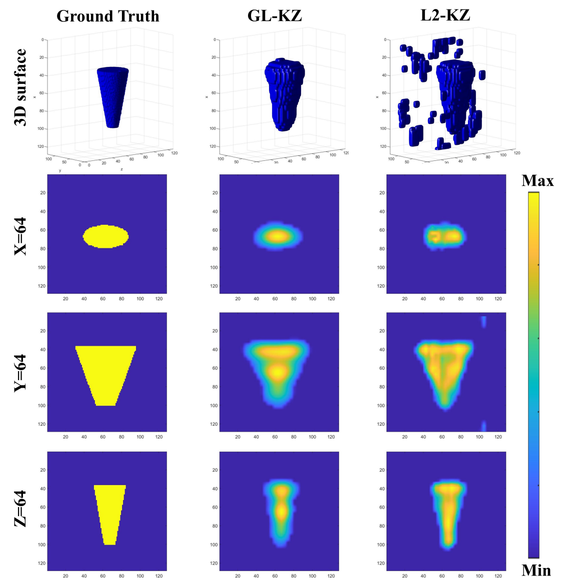

Figure 2: The 3D and slice results of L2 regularization and Gaussian weighted Laplace regularization methods.

V. Discussion

In this study, we propose a Gaussian weighted Laplace regularization based on the Kaczmarz method (GL-KZ). GL-KZ assumes that the correlation between any two voxels inside the FOV has a non-linear inverse relationship with their spatial distance. The biggest advantage of GL-KZ is that the scattered signals of the L2 regularization method is successfully alleviated and GL-KZ demonstrates better performance for accurate shape of the phantom and clean background(Figure 2). This was achieved by adopting neighborhood structural prior.

There is also a limitation of GL-KZ. The size of L regularization matrix is huge, which leads to very large memory consumption when doing matrix multiplication. In the future, we will further improve the algorithm to avoid this problem.

VI. Conclusion

In conclusion,we propose a Gaussian weighted Laplace regularization based on the Kaczmarz method for solving the inverse problem in MPI. Comparing with conventional L2 regularization method, it can improve the reconstruction accuracy. This enables MPI more suitable and practical for in vivo imaging.

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Author's statement

Conflict of interest: Authors state no conflict of interest. Informed consent: Not applicable. Ethical approval: Not applicable

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