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Magnetoviscoelastic models in the context of magnetic particle imaging

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Abstract

Some mathematical models of magnetic particle imaging include the Landau-Lifshitz-Gilbert equation that is known to model the dynamic behavior of the magnetization vector in the micromagnetic theory. Bearing in mind the fluid-structure interaction of the magnetic particles in a viscoelastic environment like blood or tissue, we discuss a modeling approach of the underlying physics that takes a magnetoviscoelastic coupling into account. In particular, we discuss applicability of models for the evolution of magnetoviscoelastic materials consisting of the incompressible Navier-Stokes equations, an evolution equation for the deformation gradient and the Landau-Lifshitz-Gilbert equation. We also consider potential implications of recent work by the authors about two-component magnetoviscoelastic materials for an advanced mathematical modeling of magnetic particles embedded into viscoelastic materials.

I. Introduction

Since the seminal article by Gleich and Weizenecker [1] there has been an increased interest in establishing magnetic particle imaging (MPI), which has a huge potential for applications in medicine and materials science [2]. Correspondingly, mathematical modeling of MPI has attracted increasing attention in recent years. Here we focus on models describing the dynamics of the suspension of magnetic particles. For related models on image reconstruction, we refer to the recent articles [3,4].

For modeling the dynamics of the magnetization, the choice of the transport of the magnetization vector is crucial, cf. [5,6] and references therein. In particular one distinguishes between so-called Brownian and Néel particles. In Brownian particles, the magnetization vector is assumed aligned to the easy axis of the particle and thus rotates together with the particle. Néel particles show a more involved evolution which is governed by the Landau-Lifshitz-Gilbert equation, a well-known equation in the context of the theory of micromagnetics.

The theory of micromagnetics takes short-range exchange interactions of the magnetization vector under the influence of an external magnetic field into account, cf. e.g. [7]. The dynamic equation governing the evolution of the magnetization is the Landau-Lifshitz-Gilbert equation. Recently, a reduced version of this was incorporated into an inverse problem for MPI in order to take relaxation effects of the magnetization into account [3].

In applications of MPI, single-domain magnetic particles interact with viscoelastic materials as for instances blood or soft tissue. We think that it is important to take the interaction of magnetic particles with a surrounding viscoelastic material into account, i.e., to consider what is also called fluid-structure interaction. Up to our knowledge this is a novel approach in MPI.

With this article we intend to initiate a discussion of suitable fluid-structure interaction mechanisms in the modeling of MPI. To this end we present two models from the theory of magnetoviscoelasticity in Sections II and III that may serve as a starting point.

II. Model in the Eulerian setting

A well-known difficulty in the modeling of coupled systems of magnetic and elastic effects is that Maxwell's theory of electromagnetism is naturally defined in Eulerian coordinates (also referred to as spatial coordinates or current configuration), while elasticity theory is based on the concept of reference configurations and hence is phrased in Lagrangian coordinates (material description). The function that maps the Lagrangian to the Eulerian coordinates is called the flow map, see, e.g., the introductions of our articles on the existence and uniqueness of solutions [8,9,10,11].

The model that the second author of this article studied in recent years is defined on an arbitrary time interval $(0,T)$ and a bounded smooth domain $\Omega \subset \mathbb{R}^d$, $d = 2,3$. The velocity of the moving material is denoted by $v : (0,T)\times\Omega \to \mathbb{R}^d$. The elastic properties are modeled by the deformation tensor in Eulerian coordinates. In linearized elasticity this would correspond to the elastic strain. While the velocity is the temporal derivative of the flow map, the deformation gradient is the spatial derivative (gradient) of the flow map with respect to the Lagrangian coordinate. The deformation gradient transformed to Eulerian coordinates is then denoted by $F : (0,T)\times\Omega \to \mathbb{R}^(d\times d)$. The magnetization is considered as a field and denoted by $M : (0,T)\times\Omega \to \mathbb{R}^3$ with $|M| \equiv M_s$ for some saturation constant $M_s > 0$, i.e. M is mapped to a ball of radius M_s.

We derive the system of partial differential equations in a variational approach from the dissipation and the total energy, which is the sum of the kinetic energy, the elastic stored energy and the micromagnetic energy

$$\int_\Omega (A|\nabla M|^2 + \psi(F,M) - \frac{1}{2}H(M)\cdot M - \mu_0 M\cdot H_{ext})\,dx\,,$$

where $A > 0$ is the exchange constant, $\mu_0 > 0$ the magnetic permeability, ψ the energy density due to anisotropy, which in our setting of magnetoelasticity depends on M and the deformation tensor F. Moreover, H_{ext} is the external magnetic field and $H(M)$ denotes the magnetic field. Without electric effects and currents, Maxwell's equations reduce to $\nabla\cdot B = 0$, $\nabla\times H = 0$. Here, B denotes the magnetic induction and satisfies $B = \mu_0(M+H)$. Furthermore, we assume that the dynamics of the magnetization vector is governed by the Landau-Lifshitz-Gilbert equation, see below.

We also take viscosity and incompressibility into account; for simplicity we assume the mass density to be constant. Then we arrive at the following system of partial differential equations that consists of (i) the incompressible Navier-Stokes equations $\nabla\cdot v = 0$ and

$$\partial_t v + (v\cdot\nabla)v = -\nabla p + \nu\Delta v + \nabla\cdot\tau + \mu_0\nabla^T H(M)M + \mu_0\nabla^T H_{ext} M$$

with the pressure p and the stress tensor

$$\tau = \partial_F W(F)F^T - 2A\nabla^T M\nabla M + \partial_F\psi(F,M)F^T\,,$$

where W denotes the elastic stored energy density. Moreover, the model consists of (ii) a transport equation for the deformation gradient

$$\partial_t F + (v\cdot\nabla)F - \nabla v F = 0$$

and finally (iii) the Landau-Lifshitz-Gilbert equation adapted to the transport $\partial_t M + (v\cdot\nabla)M$ of M:

$$\partial_t M + (v\cdot\nabla)M = -\gamma M\times H_{eff} - \lambda M\times H_{eff}\,,$$

where the effective magnetic field is given by

$$H_{eff} = 2A\Delta M - \partial_M\psi(F,M) + \mu_0 H(M) + \mu_0 H_{ext}$$

and $\gamma > 0$ is the electron gyromagnetic ratio, and $\lambda > 0$ is a damping parameter. This system is accomplished with certain boundary and initial conditions. It models magnetoviscoelastic materials and was mathematically analyzed in various special settings, see [8,9,10,11].

The transport chosen for M is based on the assumption that the magnetization exactly follows the flow of the fluid. More advanced transports like a parallel transport or a transport depending on the shape of the particles as discussed e.g. in [12] will be addressed in future work.

III. Two-component systems

Here we present a different approach which treats the magnetoviscoelastic material as a composite material consisting of two components, one being viscoelastic and the other being magnetoviscoelastic. This might have applications to clusters of magnetic particles embedded into human tissue.

In a two-component material let the components ("phases") be described by a ("order") parameter $\phi : (0,T)\times\Omega \to [0,1]$ that is the difference between the volume fractions of the two components. For sharp interface models, ϕ is supposed to satisfy a transport equation while in the diffuse interface case, i.e. the case allowing for a partial mixing of two fluids (which can model the coating of the particles in MPI), ϕ solves a gradient flow equation of Cahn-Hilliard nature. The parameter ϕ then also enters into the stress tensor of the Navier-Stokes equation and as prefactors of the terms involving F and M. We refer to [13,14] for first analytical results in a setting neglecting stress contributions from the deformation tensor. The article [13] concerns the partial mixing of two magnetic fluids with matched fluid densities and the mixing is described by a double well potential. Whereas [14] deals with fluids undergoing partial mixing and having unmatched densities; the mixing phenomenon is modeled by a singular potential.

IV. Conclusions

We presented two approaches to a mathematical modeling of magnetic particles in a viscoelastic material. For both, results from mathematical analysis about the existence of solutions are available (at least for special settings) [8,9,10,11,13,14]. However, there is a need to further discuss appropriate assumptions on the fluid-structure interaction relevant in MPI, which we wish to initiate here. When successful, it will provide a more detailed model of the dynamics of the magnetic particles under the influence of an external magnetic field in medical applications. On the long run, this might also have an impact on related inverse problems.

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Author's statement

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