International Journal on Magnetic Particle Imaging
Vol 8, No 1, Suppl 1, Article ID 2203042, 4 Pages

Infinite Science Publishing

Proceedings Article

Fast and artifact reducing joint multi-patch MPI reconstruction

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Abstract

The method of magnetic particle imaging (MPI) has a limited field of view due to physiological constraints. It is thus necessary to enlarge the field of view by a multi-patch approach in order to cover larger volumes. During reconstruction, truncation artifacts arise at the patches boundaries. We apply stochastic primal-dual hybrid gradient method to jointly reconstruct multi-patch magnetic particle images. We are thus able to apply a regularization taking into account neighborhood structures, not only on one patch but over all patches. Our experiments show that the quality of our reconstructions is significantly higher than those obtained by Kaczmarz method. Moreover, a joint reconstruction considerably reduces the computational costs compared to multiple single-patch reconstructions.

I. Introduction

The relation between the particle distribution and the received signal in magnetic particle imaging (MPI) can be described by a linear operator. We thus have to solve an ill-posed linear system to reconstruct a magnetic particle image. The discrete inverse problem is of the form

$$Sc = u,$$

where $S \in \mathbb{R}^{2K \times N}$ is the measured forward operator of MPI in the Fourier domain, $c \in \mathbb{R}^N$ is the unknown concentration to be recovered and $u \in \mathbb{R}^{2K}$ contains the Fourier coefficients of the received signal. By N we denote the number of points measured in space and K is the number of frequencies. Note that we split into the real and imaginary parts of the matrix and the data.

Due to physiological constraints, the covered field of view (FOV) in MPI is limited to a few cubic centimeters. It is thus necessary to perform multi-patch measurements to capture a larger volume. After each measurement of a certain volume the field-free point (FFP) is shifted by static focus fields. We call the measurement obtained at a certain FFP position a patch.

Reconstruction of multi-patch data is an important field of research and various approaches have been proposed. When reconstructing drive-field patches separately, artifacts at the borders occur as well as stripe artifacts, when overlapping measurements are taken [1]. Joint reconstruction of the different patches yields artifact-reduced images, but the need of storing one system matrix (SM) measured on the full FOV per patch poses huge memory requirements and is very time-consuming in the calibration process [2]. Exploiting the sparsity of that joint system matrix and reusing a single calibration scan reduces the computational effort [3]. However, to apply that approach we have to assume shift invariance of the system matrices, which holds only for ideal magnetic fields, but has to be generalized for large field imperfections. Boberg et. al. perform this generalization by applying a clustering in similar matrices, where within each cluster the shift invariance holds in good approximation [4]. All approaches mentioned solve their linear system by a Kaczmarz-type method. For a joint reconstruction approach, this means that Kaczmarz has to be performed on the huge stacked equation system. In contrast, for separate reconstruction of the patches, it means that regularization can only be applied to one patch at a time.

II. Methods

Assume that the FOV covered by the MPI scanner is divided into L different patches. We acquire one system matrix S_l per patch. The measurements for patch l are stored in u_l.

Recently, it was proposed to use stochastic primal-dual hybrid gradient (SPDHG) method for MPI reconstruction as to allow for various different regularization approaches such as fused lasso, classical Tikhonov or ℓ^1 regularization; and still have an algorithm capable of online reconstruction [5]. The numerical algorithm is very easy to implement and to use. Moreover, it was shown that the algorithm is highly competitive to the state-of-the-art, the Kaczmarz method, regarding the quality of the reconstructions as well as the computational effort when performing a single-patch reconstruction. In this paper, we use SPDHG for multi-patch reconstruction. This allows us to apply e.g. total variation regularization, which takes into account neighborhood structures of the voxels and thus reduces artifacts near the patches' boundaries. Moreover, the splitting of the stacked linear equation system into smaller systems is computationally less demanding. We compare our approach of jointly reconstructing by SPDHG with fused lasso (FL) regularization to separate reconstructions of the patches by SPDHG with FL regularization and Kaczmarz method with Tikhonov regularization and non-negativity constraint in terms of runtime and quality of reconstruction.

For the separate reconstructions, we have to solve a system of linear equations, i.e.

$$S_l c_l = u_l \quad \forall l \in \{1, ..., L\}$$

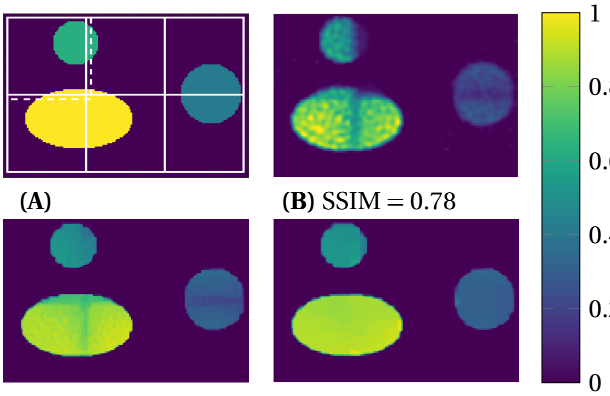
where c_l contains the l-th patch plus overlap. We apply FL regularization and solve

$$\min_{c_l \geq 0} \alpha \mathrm{TV}(c_l) + \beta \|c_l\|_1 + \|S_l c_l - u_l\|_1 \ \forall l \in \{1, ..., L\}. \quad (1)$$

The reconstructions c_l are then stacked together to form c. We use a fade-in/ fade-out approach for the overlapping parts to reduce artifacts near the boundaries.

For the joint reconstruction approach, we use the sparse linear equation

$$\begin{pmatrix} [\quad S_1 \quad] & & \\ & \ddots & \\ & & [\quad S_L \quad] \end{pmatrix} c = \begin{pmatrix} u_1 \\ \vdots \\ u_L \end{pmatrix},$$



(A) **(B)** SSIM = 0.78 **(C)** SSIM = 0.90 **(D)** SSIM = 0.92

Figure 1: The phantom (A) and a single-patch (SP) reconstruction by Kaczmarz method (B). The SP reconstruction by SPDHG with FL regularization (C) suffers from severe boundary artifacts within homogeneous regions, whereas they are resolved well for the joint approach solved by SPDHG (D).

where the matrices $S_1, ..., S_L$ only affect the part of c, which contains the corresponding patch plus overlap. We can thus also formulate the equation as

$$(S_l \circ P_l)\, c = u_l \quad \forall l \in \{1, \ldots, L\},$$

where P_l performs appropriate index mapping. When applying SPDHG to solve that equation, we perform all computations on the small matrices S_l. Still, the regularization works on c as a whole, not on the separate patches only. Moreover, we can apply additional splitting on the data as proposed in [5] using

$$S_l = \left(S_{l,1}, \ldots, S_{l,M}\right)^T, \quad (2)$$

where the matrix is divided into M submatrices by row-wise separation. Matrix-vector products within the algorithm are then performed on the small matrices $S_{l,m}$. Our approach then solves the minimization problem

$$\min_{c \geq 0} \alpha \mathrm{TV}(c) + \beta \|c\|_1 + \sum_{l=1}^{L} \sum_{m=1}^{M} \left\| S_{l,m} \circ P_l c - u_{l,m} \right\|_1. \quad (3)$$

III. Results

Numerical experiments are performed on simulated data with simulated system matrices. We simulate 2D data in the time domain using the Langevin magnetization model and add Gaussian white noise. The data are then Fourier transformed and processed as measured data. The FFP scanner is modeled as based on the real scanner at UKE Hamburg-Eppendorf with drive field excitation frequencies $f_x = 6 \cdot 10^5/24$ Hz, $f_y = 6 \cdot 10^5/25$ Hz and a repetition time of 10^{-3} s, the voxel size is $2 \times 2\mathrm{mm}^2$. Our first phantom is depicted in Figure 1 (A) and consists of large homogeneous regions. The drive-field FOV for the

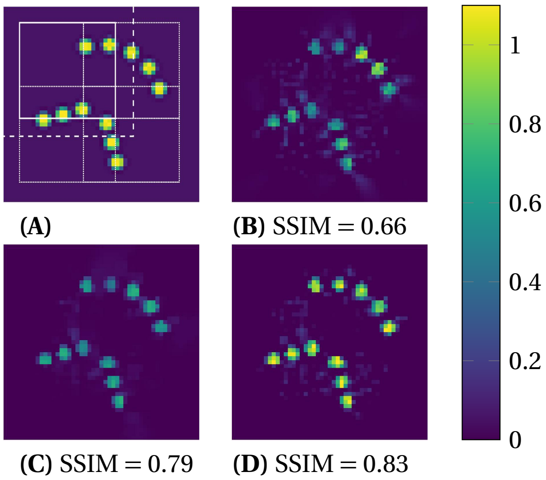


Figure 2: The phantom (A), patches and overscan indicated, and a SP Kaczmarz reconstruction (B). The SPDHG reconstruction with FL regularization (C) underestimates the concentration after 10^4 iterations per patch. It is well resolved for the joint approach solved by SPDHG after 10^4 joint iterations (D).

six different patches (solid) and overscan (dashed) for the upper left one are indicated in the figure. The SM FOV is of size 56×56 voxels, overscan is three voxels in each direction. Single-patch reconstructions then lead to severe boundary artifacts. By applying SPDHG for the joint approach the quality of reconstruction can be improved as is shown in Figure 1 (D). This is underlined by the structural similarity index measure (SSIM).

Further experiments are performed on a sparser phantom, depicted in Figure 2 (A), which is based on the phantom used in [6]. We now have an overscan of 4.5 voxels in either direction for an SM FOV of 33×33 voxels, the full reconstruction grid measures 49×49 voxels. When using SPDHG, we study the influence of additional data splitting as in (2) on run time and quality of reconstruction. We set the probability for a data step in the algorithm to $2/3$ for the single-patch reconstructions and to $11/12$ for the joint approach. By those choices we ensure that a similar amount of smoothing (once in 12 iterations in expectation) is applied. Note that our standard setting for the joint approach is the use of four data batches corresponding to the four patches and explicitly mentioned splitting means on top of that. We compare the SSIM values reached after a fixed number of epochs for single-patch approach and joint approach. Figure 3 underlines the quality enhancements of SPDHG in comparison to Kaczmarz algorithm. Moreover, it shows the potential computational savings by using the joint approach and data splitting.

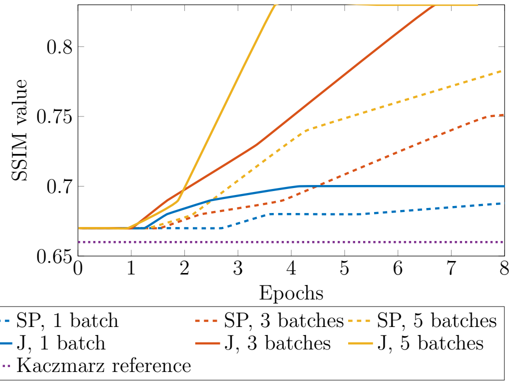


Figure 3: SSIM values reached against run time of the algorithm in epochs. The SP algorithm (dashed) is in general slower than the joint (J) approach and both outperform Kaczmarz in terms of the highest quality reached. SP SPDHG is competitive to SP Kaczmarz in terms of run time if adequately tuned [5]. Data splitting further improves the speed of SPDHG.

IV. Discussion

SPDHG algorithm can significantly improve MPI multi-patch reconstructions. The possible choice of regularization improves the visual quality of the images and computational enhancements can be employed by data splitting. Reconstruction of measured data as well as a detailed comparison to the joint Kaczmarz approach [2] in terms of run time and quality of reconstructions is left for future work.

Acknowledgments

C.B. acknowledges the support by the Deutsche Forschungsgemeinschaft (DFG) within the Research Training Group GRK 2583 "Modeling, Simulation and Optimization of Fluid Dynamic Applications".

Author's statement

The authors state no conflict of interest.

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