International Journal on Magnetic Particle Imaging
Vol 8, No 1, Suppl 1, Article ID 2203062, 4 Pages

Infinite Science Publishing

Proceedings Article

Compensating model imperfections during image reconstruction via RESESOP

Marius Nitzsche [a,*] · Hannes Albers [b] · Bernadette N Hahn [a] · Tobias Kluth [b,c]

[a] Department of Mathematics, University of Stuttgart, Stuttgart, Germany
[b] Center for Industrial Mathematics, University of Bremen, Bremen, Germany
[c] Center for Optimization and Approximation, University of Hamburg, Hamburg, Germany
[*] Corresponding author, email: marius.nitzsche@imng.uni-stuttgart.de

Abstract

To avoid the time consuming process of measuring the system function of magnetic particle imaging, model-based system matrix simulation is an alternative. However, this is a complex procedure leading to model imperfections, which influence the accuracy of the resulting system matrix as well as the quality of the image reconstruction. Standard reconstruction algorithms like regularized Kaczmarz are not able to take this inexactness into account and produce poor quality images. The RESESOP-Kaczmarz algorithm is a novel image reconstruction method, which can factor in model imperfections or dynamics. We examine and discuss the compensating characteristics of RESESOP-Kaczmarz regarding model inexactness in magnetic particle imaging.

I. Introduction

A main problem of interest in Magnetic Particle Imaging (MPI) is the determination of the system function. The measurement-based approach is the most accurate method; however, it has many drawbacks. The calibration scans consume a lot of time and each imaging sequence needs a separate acquisition. The alternative is a model-based approach. It allows the simulation of the system matrix at arbitrarily fine sampling grids and does not need the time consuming calibration procedure. A research goal of MPI is the improvement of the accuracy of the model-based approach but inaccuracies in the system function may lead to severe loss of quality in the image reconstruction.

After determining the system matrix, it is necessary to reconstruct an image from measured data. One common solver is the regularized Kaczmarz method [1], which works well if applied to a static, measurement-based case. However, reconstructing images from less perfect data or system function via Kaczmarz leads to poor results. This lack of quality can be caused by a lot of noise or motion during data measurement or model inexactness using the model-based concept.

A promising approach to deal with model imperfections is the idea to consider errors of the model in the reconstruction process. This can be done, for instance, via the RESESOP-Kaczmarz algorithm which was motivated by an application to dynamic imaging problems [2]. In this context, the method interprets the motion of the examined phantoms as model inexactness. Thus, the algorithm does only need an estimation and not the exact information of the dynamics.

The goal in this article is to interpret the less accurate determination of the system matrix using the model-based approach as a model inexactness in the RESESOP-Kaczmarz algorithm and to examine in what way it is able to compensate model imperfections.

II. Methods

We consider a fully simulated numerical experiment, where the ground truth operator (GT) is generated with the model presented in [3] as Model B3. This simulated system matrix consists of dynamic simulations of Néel-type particle magnetization dynamics, where the uniaxial particle anisotropy is dependent on the position in the field of view. While the anisotropy is very small in the center, it grows larger towards the boundaries. This is motivated by the structure of the applied field in MPI, where the static part is assumed to lead to physical rotation of the particles and affect their combined anisotropy energy landscape.

The second model operator (AO) is obtained by simulating the Néel response for a fixed anisotropy constant $K^{\text{anis}} = 625\,\text{J/m}^3$ and different orientations, then taking the average of the orientations. Both system matrices are simulated using the toolbox presented in [4].

II.I. Image Reconstruction

It is necessary to solve the linear system (here formulated in time domain)

$$Ac = v, \quad A \in \mathbb{R}^{M\times N}, c \in \mathbb{R}^N, v \in \mathbb{R}^M$$

to compute concentration c from measured data v with system matrix A. This is an ill-posed inverse problem and has to be solved accordingly by special algorithms.

A common solver in MPI is the Kaczmarz method with Tikhonov regularization. The algorithm mainly consists of a fixed point iteration. Further information can be found in the literature, see, e.g., [1].

RESESOP-Kaczmarz is an alternative solver, which can take model inaccuracies into account. It is based on the sequential subspace optimization method (SESOP) [5]. As subspaces, we consider hyperplanes and stripes ($u \in X; \alpha, \xi \in \mathbb{R}$ with $\xi > 0$):

$$H(u,\alpha) = \{x \in X : \langle u, x\rangle = \alpha\},$$
$$H(u,\alpha,\xi) = \{x \in X : |\langle u, x\rangle - \alpha| \le \xi\}.$$

Furthermore, the metric projection onto a hyperplane in Hilbert spaces can be written as

$$P_{H(u,\alpha)}(x) = x - \frac{\langle u, x\rangle - \alpha}{\|u\|^2} u.$$

The main idea of SESOP is to approximate the searched-for solution iteratively with metric projections onto intersections of hyperplanes.

However, this algorithm does not take noisy data or model imperfections into account, which we characterize in the following by levels $\delta, \eta \in \mathbb{R}$:

Noisy data:	$\|v - v^\delta\| \le \delta,$
Inexact forward operator:	$\|A - A^\eta\| \le \eta.$

It is possible to modify SESOP with a form of regularization. Instead of projecting the approximate solution onto intersections of hyperplanes, regularized SESOP (RESESOP) projects onto intersections of stripes. The width of the stripes is chosen in dependence of the level of model inexactness and noise. Morozovs discrepancy principle introduces a stopping criterion; as a result, it can be proven under certain conditions that the method is a regularization [6].

Furthermore, RESESOP can be combined with the Kaczmarz method. While RESESOP enables the usage of local properties of the inverse problem in the regularization (such as local model inexactness levels), Kaczmarz allows the algorithm to combine such local information from K subproblems, which arise for instance through multiple trajectories or dynamics, and apply it to improve the solution.

An important part of the algorithm are the search directions, which span the hyperplanes and enable convergence. A suitable choice is the product of the adjoint system matrix and the corresponding residuum. The number of search directions is not further specified; however, two are a common choice and in practice a good compromise between complexity and quality of solution.

Algorithm 1 is a general form of the RESESOP-Kaczmarz method. One full iteration consists of K subiterations, which include a metric projection onto a stripe.

Algorithm 1 RESESOP-Kaczmarz

Choose c_0 and constants $\rho > 0, \tau_k > 1$ with $k \in \mathcal{K}$. Let n be the iteration index.
while $c_n \neq c_{n-K}$ for $n \mod K = 0$ **do**
  if $\|Ac_n - v_{[n]}^{\eta,\delta}\| \le \tau_{[n]}(\eta_{[n]}\rho + \delta_{[n]})$ **then**
    $c_{n+1} = c_n$
  else
    Choose finite index set $I_n \subset \{0, 1, \ldots, n\}$
    Choose $w_{n,i}$ for all $i \in I_n$ and determine search directions $u_{n,i} = A^* w_{n,i}$
    Define $H_n^{\eta,\delta} := \bigcap_{i\in I_n} H(u_{n,i}, \alpha_{n,i}, \xi_{n,i})$
    with parameters $\alpha_{n,i} = \langle w_{n,i}, v_{[i]}^{\eta,\delta}\rangle$ and $\xi_{n,i} = (\eta_{[i]}\rho + \delta_{[i]}) * \|w_{n,i}\|$
    Compute $c_{n+1} = P_{H_n^{\eta,\delta}}(c_n)$
  end if
end while

III. Experiments

We have simulated two different system matrices GT and AO of size 61 × 61 × 5 voxels as described in section II. We used a resolution of the grid of 0.5 mm, with a 2D Lissajous excitation and anisotropy constants of up to $1250\,\text{J/m}^3$. The data was simulated on a slightly shifted 63 × 63 × 7 grid and some Gaussian noise was added.

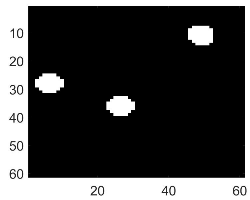
10.18416/ijmpi.2022.2203062

The three dimensional phantom consists of three cylinders. They are located in three different positions: One of it is close to the center, one close to a corner and the last close to a boundary.

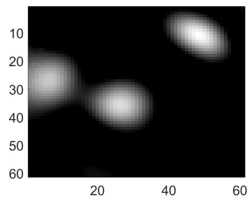
Using the regularized Kaczmarz method, images are reconstructed with GT as well as AO. Additionally, the inverse problem involving AO is solved with the RESESOP-Kaczmarz algorithm. The level of model inexactness is computed as a scaled relative error of AO compared to GT.

IV. Results

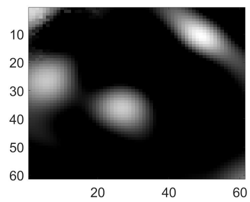
Figure 1 depicts results of the above described experiments.



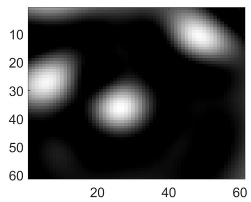
(a) Phantom



(b) GT - regularized Kaczmarz



(c) AO - regularized Kaczmarz



(d) AO - RESESOP-Kaczmarz

Figure 1: Reconstructions with different operators and methods of phantom 1a. We depict the same slice of each 3D solution.

If we compare figures 1b and 1c, the influence of model imperfections onto image reconstruction is visible. While the cylinder located close to the center is reconstructed very similarly, concentrations at boundaries and especially at corners are depicted blurred and inexact when computed with AO. Furthermore, false artifacts are visible in corners.

The image computed by RESESOP-Kaczmarz with AO shows a better reconstruction. The cylinder in the corner has a round shape and there are no distinctive false artifacts. However, the concentrations are blurrier than the ones reconstructed with GT.

The results observed in the reconstructions can be understood if we examine the relative error of AO compared to GT, which is shown in figure 2. At the center of the field of view, there is almost no difference between the operators, whereas the errors are larger at the boundaries, especially in the corners. Therefore, all three reconstructions show comparable results at the center of the images.

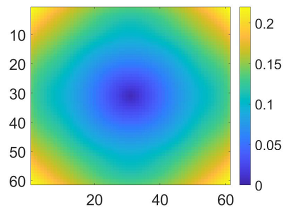


Figure 2: One slice of the relative error of AO compared to GT.

However, this is not the case at boundaries and corners, where the model imperfections heavily influence the quality of the image. RESESOP-Kaczmarz includes model inexactness in its reconstruction and hence reduces its influence. This shows the advantage of RESESOP-Kaczmarz compared to regularized Kaczmarz.

V. Conclusion

The RESESOP-Kaczmarz method is a promising algorithm to compensate model imperfections. Our numerical results illustrate its potential in improving the model-based reconstruction approach in MPI.

Since the approach only relies on rough estimates instead of exact information regarding the model inexactness, it can further be applied in various cases. This includes the reconstruction of dynamic phantoms or a robust reconstruction method with fully measured system matrices. Furthermore, ongoing work is the validation on real data as well as the transfer of the algorithm onto frequency space.

Acknowledgments

Authors acknowledge the support by 'Deutsche Forschungsgemeinschaft' (project 426078691).

Author's statement

Authors state no conflict of interest.

References

[1] T. Knopp and T. M. Buzug, Magnetic Particle Imaging: An Introduction to Imaging Principles and Scanner Instrumentation. Berlin/Heidelberg: Springer, 2012, doi:10.1007/978-3-642-04199-0.

[2] S. Blanke, B. Hahn, and A. Wald. Inverse Problems with inexact forward operator: Iterative regularization and application in dynamic imaging. *Inverse Problems*, 36, 2020, doi:10.1088/1361-6420/abb5e1.

[3] T. Kluth, P. Szwargulski, and T. Knopp. Towards accurate modeling of the multidimensional magnetic particle imaging physics. *New Journal of Physics*, 21(10):103032, 2019, doi:10.1088/1367-2630/ab4938.

10.18416/ijmpi.2022.2203062

[4] H. Albers, T. Kluth, and T. Knopp. Simulating magnetization dynamics of large ensembles of single domain nanoparticles: Numerical study of Brown/Néel dynamics and parameter identification problems in magnetic particle imaging. *Journal of Magnetism and Magnetic Materials*, 541:168508, 2022, doi:https://doi.org/10.1016/j.jmmm.2021.168508.

[5] F. Schöpfer, T. Schuster, and A. Louis. Metric and Bregman projections onto affine subspaces and their computation via sequential subspace optimization methods. *J. Inv. Ill-Posed Problems*, 15:1–29, 2007, doi:https://doi.org/10.1515/JIIP.2008.026.

[6] F. Schöpfer and T. Schuster. Fast regularizing sequential subspace optimization in Banach spaces. *Inverse Problems*, 25:015013, 2009, doi:10.1088/0266-5611/25/1/015013.

