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Multi-color Kaczmarz Method for Color Magnetic Particle Imaging

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Abstract

Color magnetic particle imaging (cMPI) is an advanced technology to distinguish different kinds of nanoparticles or environment properties, such as temperature or viscosity. This technology is realized based on the different harmonic responses and relaxation behavior of particles. However, the commonly used MPI Kaczmarz method has artifacts when it is applied to cMPI reconstruction. To address this problem, a multi-color Kaczmarz method (MKZ) is proposed for cMPI reconstruction. We introduce a new interference item to identify the artifacts caused by the interference between different particles. This method showed better performance than the Kaczmarz method in numerical experiments. It can eliminate the artifacts and efficiently improve the image quality of cMPI.

I. Introduction

Magnetic particle imaging (MPI) is a novel molecular imaging technology, which can non-invasively image the super-paramagnetic iron oxide (SPIO) nanoparticle with high sensitivity and resolution [1]. Based on this technology, color magnetic particle imaging (cMPI) was proposed to distinguish different kinds of nanoparticles [2]. It can be applied to track different types of SPIO particles or identify the characteristics of their environment, and shows advantages for many biomedical applications. However, the imaging quality of cMPI is limited by the performance of reconstruction methods.

To date, many methods have been proposed for cMPI reconstruction, such as the system matrix method [3], x-space method [4], and TAURUS method [5]. The system matrix reconstruction method was applied in cMPI based on the differences in harmonic responses of different SPIO particles [3]. Previous study has proved that this method can distinguish different particle types with the row-action Kaczmarz method [3]. However, also results out the problem that not all signals can be assigned correctly by the classical Kaczmarz method.

To address this problem, a multi-color Kaczmarz method (MKZ) is proposed herein. A new interference item is introduced in Kaczmarz method to describe the artifacts caused by the interference of different particles. To verify the proposed method, we simulated the numerical experiments and compared the MKZ method with the Kaczmarz method.

II. Method

II.I. Problem Formulation for cMPI

For MPI technology, it is important to model the relationship between the particles' concentration c and the measured voltage u. The scalar magnetization of SPIO in response to the applied field $\mathbf{H}(x,t)$[A/m] obeys the following equation:

$$\mathbf{M}(x,t) = mc(x)\mathscr{L}(\beta_1\mathbf{H}) * \frac{1}{\tau}exp(-\frac{t}{\tau})\hat{u}(t) \quad (1)$$

where $\beta := \frac{u0m}{k_B T}$. k_B denotes the Boltzmann constant, m denotes the magnetic moment of the particle, T denotes the particle temperature. τ denotes the relaxation time constant of c. $\mathscr{L}(\cdot)$ and $\hat{u}(t)$ denotes the Langevin function and the Heaviside function, respectively.

Based on the magnetization response of SPIO, the measurement signal u of two different kinds of particle c_1 and c_2 can be described as follows:

$$\begin{aligned} u = u_1 + u_2 &= \frac{d}{dt}\int_\Omega B\mathbf{M}_1\,dx + \frac{d}{dt}\int_\Omega B\mathbf{M}_2\,dx \\ &= \int_\Omega s_1(x,t)c_1(x)dx + \int_\Omega s_2(x,t)c_2(x)dx \end{aligned} \quad (2)$$

where c_1 and c_2 denote the density of two different types of particles.

As the signal $u(t)$ and kernel $s(r,t)$ are both periodic continuous sequences, they can be expanded into Fourier series and discretized, and formulate the linear equation system of cMPI as follows:

$$u = S_1c_1 + S_2c_2 = (S_1\ S_2)\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \quad (3)$$

where S_1 and S_2 denote the system matrix of c_1 and c_2, respectively.

II.II. The Multi-color Kaczmarz Method for cMPI Reconstruction

To solve equation (3), it is commonly converted to the least-squares problem as follows:

$$c_c = \mathop{\mathbf{argmin}}_{c_c} ||S_c c_c - u||_2^2 + \lambda||c_c||_2^2 \quad (4)$$

where c_c denotes the concatenated individual vector of c_1 and c_2. S_c denotes the system matrix that S_1 and S_2 concatenated along the spatial axis. λ is the parameter of the regularization item.

To solve (4), we introduce an interference item $c_a \in \mathbb{R}^{N\times1}$ in Kaczmarz method [6] to describe the artifacts in cMPI results and propose the multi-color Kaczmarz method (MKZ) herein.

To describe the artificial particle distribution in results, the reconstruction result of two different types of particles c_1 and c_2 can be formulated as follows:

$$\begin{cases} c_1 = & c_{r,1} + c_{a,1} \\ c_2 = & c_{r,2} + c_{a,2} \end{cases} \quad (5)$$

where $c_r \in \mathbb{R}^{N\times1}$ denotes the real particle distribution. Subsequently, equation (3) can be further modified as follows:

$$\begin{aligned} u &= S_1c_1 + S_2c_2 \\ &= S_1\left(c_{r,1} + c_{a,1}\right) + S_2\left(c_{r,2} + c_{a,2}\right) \\ &= S_1c_{r,1} + S_2c_{r,2} + \eta \end{aligned} \quad (6)$$

where $\eta = S_1c_{a,1} + S_2c_{a,2}$ denotes the interference item that leads to the artifacts. Based on the relationship between c_a and c_r, $c_{a,1}$ and $c_{a,2}$ can be approximated by the $c_{r,2}$ and $c_{r,1}$, respectively.

To acquire real particle distribution c_r, we modify the Kaczmarz method and formulate the MKZ based on equation (6). The iteration equation of MKZ is revised as follows:

$$\begin{aligned} \beta &= \frac{\langle s_i, c_k\rangle + \sqrt{\lambda}z_i + \eta_i - u_i}{||s_i||_2^2 + \lambda} \\ c_{k+1} &= c_k - \beta s_i \\ z_i &= z_i - \beta\sqrt{\lambda} \end{aligned} \quad (7)$$

where η_i denotes ith elements in interference item η.

III. Experiments and Results

To verify the effectiveness of the proposed method, we performed numerical experiments herein. Based on the simulation data, we compared the performance of the proposed method and the classical Kaczmarz method. All reconstructions were performed on a computer with an Intel® Core™ i7-6700K CPU (4.00 GHz) and 64 GB RAM.

III.I. Experiments Set

To formulate the 2D system matrix and the measurement signal for experiments, we numerically simulated data based on the mathematical theory of the MPI imaging process. The gradient field was set to 1.5 T/mμ_0 and the excitation field was set to 14 mT/μ_0 in both x- and y-directions. The excitation field frequency was set to 25 kHz in the x-direction and 24.75 kHz in the y-direction. The full field-of-view (FOV) was discretized at 38 × 38 grid points with a 1 mm distance step. The sampling frequency was set to 2.5 MHz and the sampling time was 4 ms.

The relaxation time constant τ for two targets was set to 2.67μs and 4.15μs to simulate the Nanomag-MIP and VivoTrax SPIOs, respectively [5]. The Gaussian noise was

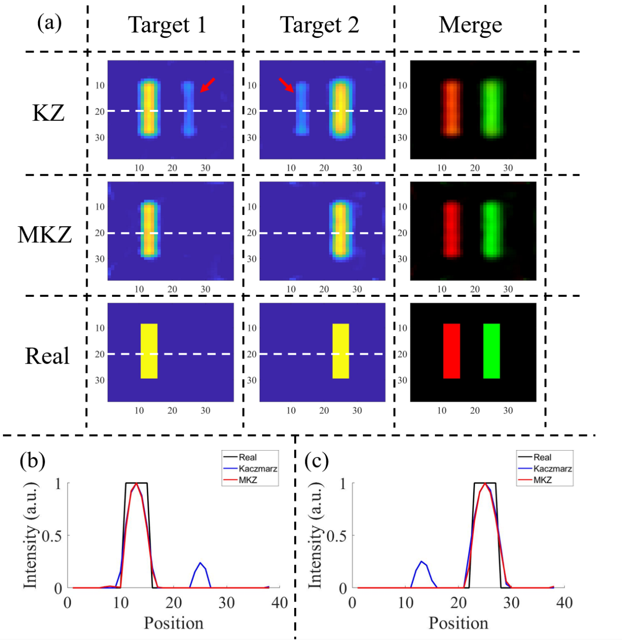

Figure 1: The reconstruction result in 40 dB noise level. (a): The white dot lines represent the profile lines. The red arrows point out artifacts in results. (b) and (c): The profile lines of target 1 and target 2.

added by the *awgn* function in Matlab. The regularization parameter λ was set to 10^{-3} for both methods, and the iteration number was set to 50. All the reconstruction results were normalized before evaluation.

III.II. Numerical Results

The reconstruction results of the Kaczmarz and MKZ method in 40 dB noise level were performed in Fig.1.

There were obvious artifacts was observed when using the Kaczmarz method, as we pointed by red arrows. On contrary, the MKZ method provided better visualization of results in the separate results. It successfully eliminated the artifacts and accurately reconstructed the real particle distribution. This conclusion is also validated by the profile lines of images.

To further validate the robustness of the proposed method, five different noise levels: 30 dB, 35 dB, 40 dB, 45 dB, 50 dB were tested. To quantify the quality of results, the signal-to-background ratio (SBR) was computed as follows [7]:

$$\mathbf{SBR} = \frac{\sum c_t}{\sum c_i} \tag{8}$$

where c_t and c_i denote the region of true result and the interference item, respectively.The evaluation indicators of results in different noise levels were performed in Fig.2.

Based on these results, it can be concluded that the MKZ can robustly improve the quality of cMPI reconstruction results. In different noise levels, the MKZ can

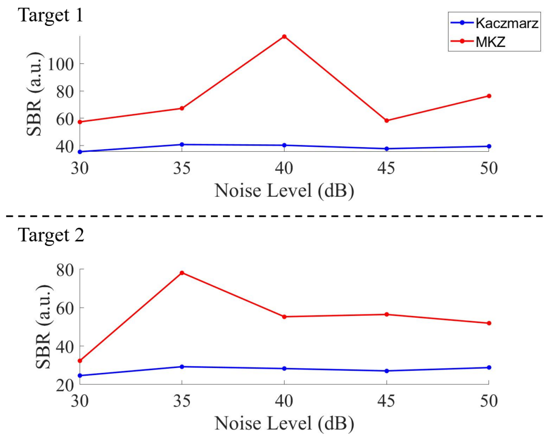

Figure 2: The evaluation indicators of reconstruction results in different noise levels.

improve the SBR indicator of the two targets by at least 25%. Additionally, the MKZ can improve the SBR indicator to 120 for target 1 at 40 dB noise level. In comparison, the SBR indicator of the Kaczmarz method was only 40 in the same condition.

To sum up, the MKZ method can eliminate the artifacts caused by the interference between two different targets, and significantly improve the reconstruction quality. In the same iteration parameters and noise condition, it showed better performance than the classical Kaczmarz method.

IV. Conclusion

In this paper, we present an advanced MKZ method for cMPI reconstruction. We introduce an interference item to describe the artificial target in results and embedded it in the MKZ method. By applying a modified iteration equation, the MKZ can eliminate the artifacts in the results. This improvement method can be widely used in cMPI technology to improve the reconstruction quality.

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Author's statement

Conflict of interest: Authors state no conflict of interest.

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