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Multi–dimensional Debye model for nanoparticle magnetization in magnetic particle imaging

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Abstract

Magnetic particle imaging (MPI) is a new medical modality to safely image the concentration distribution of superparamagnetic iron-oxide nanoparticles (SPIOs). It relies on the nonlinear magnetization response of SPIOs under a time-varying magnetic field to induce an output voltage signal. When the magnetic field is multidimensional, the accuracy of the first-order Debye model decreases in describing the magnetization process. To solve this problem, we propose a multi–dimensional Debye model, which considers each dimensional magnetic field's contribution to the magnetization of SPIOs. Through various experiments, the proposed multi–dimensional Debye model shows superiority over the first-order Debye model, with a 30% lower root-mean-square error in modeling the magnetization. The multi–dimensional Debye model can accurately analyze the influence of different magnetic fields on the SPIOs. This model can further guide MPI instrument optimization.

I. Introduction

Magnetic particle imaging (MPI) is a promising modality that has been applied in various medical applications [1], such as cardiovascular imaging [2]. Accurately modeling the magnetization behavior of SPIOs is essential for MPI signal analysis and instrument improvement. Compared with the current models for MPI [1, 3], the first-order Debye model is the most concise for describing the magnetization relaxation effect of the SPIOs [4]. The first-order Debye model simplified the magnetization process as an exponential decay with a relaxation time constant, τ [4]. However, a single relaxation time cannot sufficiently describe the dynamic magnetization under multidimensional magnetic fields [3]. This problem would cause MPI theory analysis inaccuracy and leads to image artifacts in the reconstruction process. Thus, it is necessary to establish an accurate magnetization model for multidimensional MPI.

In this study, a multi–dimensional Debye model is developed to describe the magnetization of SPIOs under multidimensional magnetic fields in MPI. The exponential decay is a superposition of several first-order Debye terms. Each term owes one relaxation time constant, which reflects the relaxation behavior caused by a one-dimensional magnetic field. To further evaluate the contribution of a specific magnetic field component, the magnetic field strength coefficient is added to each first-order Debye term. Furthermore, the accuracy of the multi–dimensional Debye model is validated by comparing it with actual magnetization measurement data. Its robustness is also validated through various frequency and amplitude experiments.

II. Theory and methodology

The signal-generating process of the MPI theory is presented here. In a multidimensional MPI, the gradient field is in the form. $\boldsymbol{H}(\boldsymbol{x})=\boldsymbol{G}\boldsymbol{x}$, where $\boldsymbol{G}$ is the gradient matrix, and $\boldsymbol{x}=[x\ y\ z]^T$ denotes the position in real space [5]. The time-varying excitation magnetic fields are $\boldsymbol{H}_s(t)=[H_x(t)\,H_y(t)\,H_z(t)]^T$. The total effective magnetic field can be described as $\boldsymbol{H}(t,\boldsymbol{x})=\boldsymbol{H}_s(t)-\boldsymbol{G}\boldsymbol{x}$.

The magnetization of SPIOs $\boldsymbol{M}$ in response to the applied magnetic field can be described as the first-order Debye model [4]:

$$\boldsymbol{M}_{1st}=m\rho(\boldsymbol{x})\mathscr{L}\left(\beta\boldsymbol{H}\right)*r(t), \tag{1}$$

with

$$r(t)=(1/\tau)exp(-t/\tau)u(t), \tag{2}$$

where $\beta:=\frac{\mu_0 m}{k_B T}$, k_B is the Boltzmann constant, $m[A\cdot m^2]$ is the magnetic moment of the SPIOs, T is the particle temperature, $\mathscr{L}(\cdot)$ is the Langevin function [1], τ is the relaxation time constant, $\rho(\boldsymbol{x})$ denotes the concentration distribution of the SPIOs, $u(t)$ is the Heaviside function [4].

To compensate for the limitation of the first-order Debye model under multidimensional magnetic fields, the multi–dimensional Debye model is adopted. The multi–dimensional Debye model inherits the time domain convolution form:

$$\boldsymbol{M}_{n-ord}=m\rho(\boldsymbol{x})\mathscr{L}\left(\beta\boldsymbol{H}\right)*h(t), \tag{3}$$

with

$$h(t)=u(t)\sum_{i=1}^{n}\frac{H_i}{\|\boldsymbol{H}\|}(1/\tau_i)exp(-t/\tau_i), \tag{4}$$

where H_i is the amplitude of the i-th dimensional magnetic field component, τ_i is the relaxation time constant caused by the i-th magnetic field component, $n(\leq 3)$ is the number of magnetic field components in different directions, and $\|\boldsymbol{H}\|$ is the total effective magnetic field amplitude. This superposition form of multi–dimensional Debye model has been used in the study of magnetic permeability [6].

Via the reciprocity theorem, the induced voltage is given by

$$s(t)=\frac{d}{dt}\int B_1\boldsymbol{M}_{n-ord}(\boldsymbol{x},t)\,d\boldsymbol{x}, \tag{5}$$

where B_1 denotes the receive coil sensitivity [5].

For the parameter selection in the multi–dimensional Debye model, the first dimensional relaxation time constant τ_1 keeps consistent with that of the first-order Debye model. This parameter is estimated by the data fitting algorithm in [4]. When $n=1$, the multi–dimensional Debye model becomes the first-order Debye model. As a multi–dimensional field is added, the corresponding relaxation time constant is calculated through a nonlinear least-squares optimization method.

III. Experiments

To evaluate the performance of the multi–dimensional Debye model, the simulated magnetization curves were compared with the measurement data obtained by two-dimensional magnetic particle spectrometry (MPS). The calibrated voltage signal is integrated over time to obtain the magnetization. The robustness of the multi–dimensional Debye model was also tested under various test conditions, such as excitation frequencies and amplitudes.

For a quantitative analysis of the proposed model performance, the root-mean-square error (RMSE) was applied to calculate the error between the simulated data and the measurement data [7]:

$$RMSE=\sqrt{\frac{\sum_{i=1}^{N}(M_{real}(i)-M_{model}(i))^2}{N}}, \tag{6}$$

where M_{real} is the magnetization measured by the MPS, N is the number of data, and M_{model} is the magnetization simulated by the model.

A commercial magnetic nanoparticle, Synomag (Micromod GmbH, Germany), coated with dextran (surface: NH2), was used for testing. The iron concentration is 0.5mg/ml, and the hydrodynamic diameter is 70nm. A volume of 100 μl sample was utilized.

Two groups of experiments were conducted to validate the multi–dimensional Debye model and test its robustness:

Group 1: The x-direction excitation magnetic field was set at 10 kHz, 10 mTμ_0^{-1}. The frequency of the y-direction excitation magnetic field was set to 20 Hz, and the amplitudes were set to 0, 3, 5, 8 mTμ_0^{-1}, respectively. The receive coil is located in the x-direction.

Group 2: The x-direction excitation magnetic field was set at 10 kHz, 10 mTμ_0^{-1}. The frequency of the y-direction excitation magnetic field was set at 1 kHz, and the amplitudes were set to 0, 1, 3 mTμ_0^{-1}, respectively. The receive coil is located in the x-direction.

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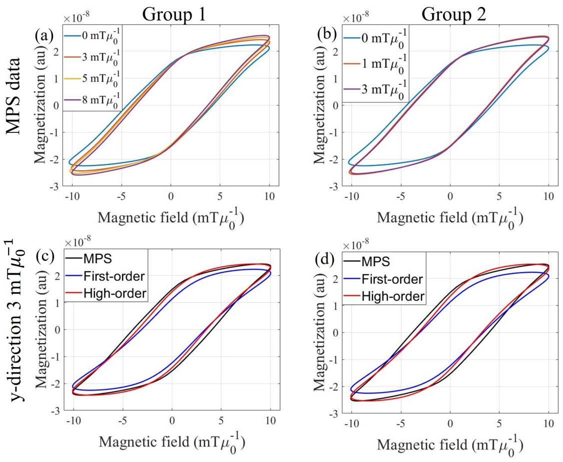

Figure 1: Magnetization curves of the multi–dimensional Debye validation experiments. The first row is MPS measurement data: (a) Group 1 (y-direction 20 Hz); (b) Group 2 (y-direction 1 kHz); The second row is simulated data by first-order Debye and multi–dimensional Debye models: (c) Group 1 (y-direction 20 Hz 3 mTμ_0^{-1}) ; (d) Group 2 (y-direction 1 kHz 3 mTμ_0^{-1}).

Table 1: The RMSE of experiments results

Group 1	unit: mTμ_0^{-1}	0	3	5	8
	first-order	0.14	0.17	0.19	0.19
	multi–dimensional	0.14	0.13	0.12	0.12
Group 2	unit: mTμ_0^{-1}	0	1	3	
	first-order	0.14	0.18	0.18	
	multi–dimensional	0.14	0.13	0.12	

IV. Results

The results of the experiments demonstrated the accuracy and broad applicability of the proposed multi–dimensional Debye model. Figure 1 shows the magnetization curves measured by the MPS instrument and calculated by the two models. With increasing the amplitude of the y-direction magnetic field, the magnetization curve of the SPIOs rises at the maximum points, as shown in Figure 1(a) and (b). In Figure 1(c) and (d), the multi–dimensional Debye model fits well with the MPS measurements in the presence of multidimensional magnetic fields. However, the first-order Debye model ignored the influence of the second-dimensional (y-direction) magnetic field, leading to inaccurate results.

The RMSE of these experiments is listed in Table 1. The average improvement of the multi–dimensional Debye model to the first-order one is 30.7%.

Two relaxation time constants of the multi–dimensional Debye model are 8.2 μs, 7.3 μs for Group 1, and 8.2 μs, 0.1 μs for Group 2. The single relaxation time constants of the first-order Debye model are 8.2 μs for both groups of experiments.

V. Discussion

The proposed multi–dimensional Debye theory can be applied to multidimensional excitation. In this work, two-dimensional excitation MPS experiments preliminarily verified the model's accuracy. Our research team is also developing three-dimensional excitation MPS. The model's performance under three-dimensional excitation will be tested in future research.

Compared to the first-order Debye model, the multi–dimensional Debye model considers the relaxation effect caused by each dimensional magnetic field. The multi–dimensional Debye model better agrees with the measurement data through more relaxation time constants. This model can be used for the optimization of excitation trajectory [8].

VI. Conclusion

In this study, we proposed an accurate and robust multi–dimensional Debye model to describe nanoparticle magnetization under multidimensional magnetic fields in MPI. The multi–dimensional Debye model considers the influence of each dimensional magnetic field component on the magnetization process. The multi–dimensional Debye model performs better through the experiments than the first-order Debye model. The multi–dimensional Debye model also has the potential to be extended to MPI instrument optimization.

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Author's statement

Authors state no conflict of interest. Informed consent has been obtained from all individuals included in this study. Conflict of interest: Authors state no conflict of interest. Informed consent: Not applicable. Ethical approval: Not applicable.

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