International Journal on Magnetic Particle Imaging
Vol 10, No 1, Suppl 1, Article ID 2403005, 5 Pages

Infinite Science Publishing

*Proceedings Article*

# Frequency components selected based on gravitational search algorithm for magnetic particle imaging reconstruction

Wenjing Jiang[a] · Shihao Shan[a] · Chenglong Zhang[a] · Yang Du[b] · Qiyuan Cheng[c] · Xiaopeng Ma[a,*]

[a]The School of Control Science and Engineering, Shandong University, Jinan, China
[b]CAS Key Laboratory of Molecular Imaging, Institute of Automation, Beijing, China
[c]Medical Engineering Department, Shandong Provincial Hospital affiliated to Shandong First Medical University, Jinan 250021, China
[*]Corresponding author, email: xiaopeng.ma@edu.edu.cn



**Abstract**

In Magnetic Particle Imaging (MPI), we often utilize system matrix to reconstruct, but it is time-consuming and memory-intensive. Therefore, use Signal-to-Noise Ratio (SNR) to reduce frequency components to speed up and reduce memory, but only use SNR values do not contain other information about the frequency and may lose crucial information required for reconstruction. To address this limitation, the frequency components selection based on gravitational search algorithm (GSA) method is proposed herein, this method leverages Newton's Law of Universal Gravitation and the Law of Kinematics to intelligently select frequency components, potentially enhancing the reconstruction image quality in MPI. Experimental results demonstrate favorable quantitative indices for the reconstructed images.

## I. Introduction

Magnetic particle imaging (MPI) is a recent molecular imaging technology that provides high spatial and temporal resolution without the use of ionizing radiation [1].

To avoid prolonged reconstruction time when using the whole system matrix for MPI reconstruction, it is necessary to select the appropriate frequency components. The common method is based on Signal-to-Noise Ratio (SNR), which can select the frequency components with lower noise but may lose crucial information for reconstruction. Therefore, we propose a novel method based on gravitational search algorithm (GSA), which can select frequency components based on Newton's Law of Universal Gravitation and the Law of Kinematics [2] to improve the quality of reconstruction image.

To assess the proposed GSA method, simulation experiments were conducted, comparing the GSA algorithm with the SNR algorithm. The findings demonstrate that the quality of reconstruction image can be prominently improved by GSA algorithm.

## II. Material and methods

### II.I. System matrix reconstruction method in MPI

The reconstruction in MPI is retrieving particle concentrations from the measured voltage. The connection between the particle concentration and the measurement signal, as established in prior research [3], can be described as follows:

$$Sc = u \tag{1}$$

where $S$ is the system matrix, $u$ is measured voltage signal, $c$ is particle concentration. To ensure the reconstructed image's quality while minimizing the reconstruction time, not all frequency components are utilized. Therefore, it is necessary to select the frequency components to reconstruction [4]. This paper introduces two distinct methodologies for frequency components selection, one based on SNR values and the other leveraging the GSA algorithm.

## II.II. Frequency selected based on SNR method

The SNR characteristic is proposed to measure the noise present in the frequency components of the system matrix [5]. To determine the SNR of a particular frequency, utilize the following formula:

$$SNR(f_i) = \frac{\sqrt{(S^{\delta}_{1,f_i})^2 + (S^{\delta}_{2,f_i})^2 + \ldots + (S^{\delta}_{N,f_i})^2}}{\sqrt{(S^{0}_{1,f_i})^2 + (S^{0}_{2,f_i})^2 + \ldots + (S^{0}_{N,f_i})^2}} \tag{2}$$

where $i = 1, 2 \ldots, M$, $S^{\delta}$ and $S^{0}$ denote the frequency component of the calibration signal with and without the sample probe, respectively, and $N$ is the number of calibration points. Arrange the frequency components in ascending order according to the magnitude of their SNR values. Establish an SNR threshold contingent upon the predetermined count of frequency components to be selected, selectively retaining those frequency components surpassing the established SNR threshold. A reduced linear system by removing some frequency components can be:

$$S_{red}\, c = u_{red} \tag{3}$$

Solving the inverse problem in (3) can obtain particle concentration.

## II.III. Frequency selected based on GSA search algorithm

The gravitational search algorithm presents fresh perspective in comparison to several heuristic algorithms based on biological behavior [2]. In a d-dimensional space, particles exert force on each other as described by Newton's Law of Universal Gravitation and the Law of Kinematics. The GSA algorithm computes the gravitational forces acting between individual particles. Particles move towards one another under the influence of gravitational forces. Notably, particles with greater mass exhibit a heightened gravitational field, thereby exerting a stronger force on neighboring particles. This

**Algorithm 1** Frequency Selection based on GSA

1. **Initialization:** Combination of frequency components based on the system matrix.
2. **For** iteration = 1 **do**
   (a) Calculate the fitness of each agent by Eq. (12).
   (b) Update the gravitational constant $G$, best and worst frequency components.
      i. Calculate $G$ by Eq. (4).
      ii. $\text{best}_j(t) = \min_{j \in \{1,2,\ldots,N\}} \text{fit}_j(t)$
      iii. $\text{worst}_j(t) = \max_{j \in \{1,2,\ldots,N\}} \text{fit}_j(t)$
   (c) Calculate $M$ and $a$ for each agent.
      i. Calculate $M$ by Eq. (5) and Eq. (6).
      ii. Calculate $a$ by Eq. (7),Eq. (8)and Eq. (9).
   (d) Update the velocity by Eq. (10).
   (e) Update the position by Eq. (11).
3. **If** the fitness does not change for five consecutive times **end the iteration.**

**Return:** Selected frequency components.

Algorithm 1

mechanism aims to delineate the pivotal roles within the particle group within a system. In the context of MPI reconstruction, this algorithm serves the purpose of searching frequency components which have better reconstruction outcomes. The accompanying pseudocode is depicted in Algorithm 1.

In our experiments, the system matrix contains a total of 1634 frequency components, with 1528 frequency components after filtering frequency under 80kHz. We choose 10% of the frequency components for GSA algorithm to reconstruction. During the initialization phase, a set of 1528 frequency components is randomly allocated into 50 agents, with each agent comprising 163 frequency components. The formulation for computing fitness is tailored to address the specific practical context under consideration. In the context of MPI frequency component selection, fitness is computed as the normalized root mean square error (NRMSE), which is calculated by Eq. (12). The Ground Truth (GT) is the phantom image we set in the experiment, and the $outcome$ is the reconstruction image of each agent.

Consequently, the fundamental steps of the GSA algorithm involve the following steps:

1. Computing the fitness for each agent, identifying the agent with the best and worst fitness.
2. Calculating the mass and acceleration of each agent.
3. Determining the velocity for the subsequent movement as the sum of a fractional component of the prior velocity and the current acceleration.
4. Update both the velocity and position of the agent.
5. The termination criterion is established as the absence of fitness variation over five consecutive iterations.

10.18416/ijmpi.2024.2403005

6. The conclusion of the algorithm involves returning the final selected combination of frequency components based on the positional attributes of the updated optimal solution.

The mathematical expressions for computing the variables employed during the iterative process are articulated as follows. The gravitational constant utilizing this equation:

$$G(t) = G(t_0) \cdot \exp{-\beta} \cdot \frac{t}{\max iter} \tag{4}$$

where $t$ is the iteration at this time, $G(t_0)$ is the gravitational constant of the first cosmic quantum time interval $t_0$ and it is initial value, $\beta$ is a constant, $\max iter$ is the maximum number of iterations [6]. Then calculate the mass $m$ and acceleration $a$ of each frequency component, using the following equations:

$$m_i(t) = \frac{fitness_i(t) - worst(t)}{best(t) - worst(t)} \tag{5}$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \tag{6}$$

where $fitness_i(t)$ is the best fitness value of agent $i^{th}$ at time $t$, $M_i(t)$ is the inertial mass of $i^{th}$ frequency component.

$$F_{ij}^d = G(t)\frac{M_i \times M_j}{||X_i(t), X_j(t)||_2}(x_j^d(t) - x_i^d(t)) \tag{7}$$

$$F_i^d(t) = \sum_{j=1, j\neq i}^{N} rand_i F_{ij}^d(t) \tag{8}$$

$$a_i^d(t) = \frac{F_i^d(t)}{M_i(t)} \tag{9}$$

$X_i$ and $X_j$ are the position vectors of agent, $x_i^d$ and $x_j^d$ are the $d^{th}$ element in $X_i$ and $X_j$, $rand_i$ is a random number among [0,1]. Use the velocity and location to describe the search strategy for this concept, using the following equations [6]:

$$v_i^d(t+1) = rand_i \cdot v_i^d(t) + a_i^d(t) \tag{10}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{11}$$

where $x_i^d$ represents the position of $i^{th}$ frequency component in the $d^{th}$ dimension, $v_i^d$ is the velocity, $a_i^d$ is the acceleration. Calculate the fitness using the following equation:

$$fitness = \frac{\sqrt{\sum(GT - outcome)^2}}{\sqrt{GT^2}} \tag{12}$$

During the iteration, the objective function is continuously reducing the fitness value.

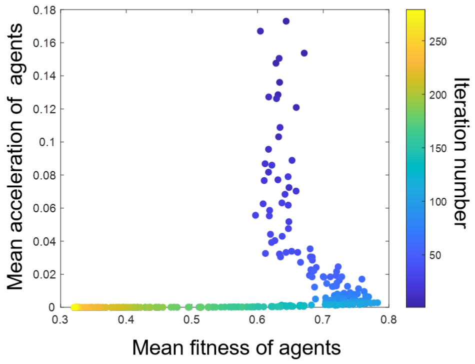


**Figure 1:** The mean fitness of agents and the mean acceleration of agents in iterations. The horizontal axis denotes the mean fitness of 50 agents in each iteration, while the vertical axis illustrates the mean acceleration of 50 agents in each iteration. The color map is the iteration number. In the advanced stages of the iteration, the fitness steadily diminishes, acceleration is almost zero, indicating a state of consistent convergence.

The GSA algorithm entails an iterative process involving the assessment of agents' fitness and acceleration variations, the acceleration of the 163 frequency components within each agent was averaged, following which the average acceleration and fitness values across the 50 agents were computed, as depicted in Fig 1.

In the initial iteration, both the fitness and acceleration values of the agents are substantial. Throughout successive iterations, GSA algorithm refines its search for optimal frequency components, resulting in a gradual decline in agents' fitness alongside a corresponding decrease in acceleration. Notably, when the acceleration converges to zero, and the fitness stabilizes at approximately 0.3. Meanwhile, the fitness value remains constant for five consecutive iterations, this signifies the identification of optimal frequency components for reconstruction, prompting the termination of the iteration process.

## III. Experiments and results

To assess the efficacy of the proposed method, we conducted simulation experiments in this study. Initially, using GSA method, we selected frequency components on the phantom of the letter 'S' and observed the reconstruction results. Subsequently, we reconstructed other phantom letters 'D, U' using the frequency components chosen by GSA algorithm on letter 'S' to confirm the proposed method's generalization performance.

Calibration data form the simulation system matrix. The field-of-view had a size of 12 mm × 12 mm, and the size of the gridding space was 19 × 19. 10% of the frequency components were chosen for GSA algorithm reconstruction, while 10% and 30% of the frequency com-

10.18416/ijmpi.2024.2403005

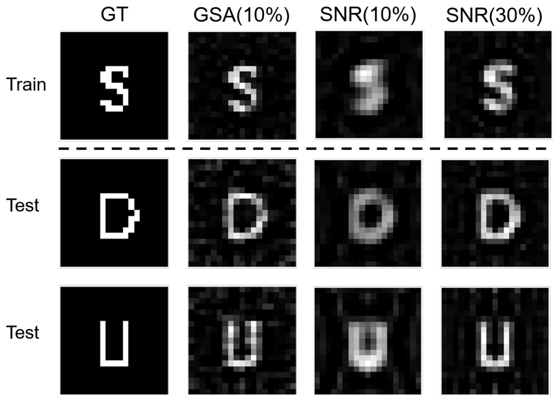


**Figure 2:** The results of the simulation experiment using the GSA algorithm and SNR method to select frequency components to reconstruction, the letter 'S' is used to select frequency components by GSA algorithm and SNR method, the letter 'D' and 'U' are used to test the selected frequency components' efficacy.

ponents were chosen for SNR reconstruction for comparison purposes. We used regularization parameter $\lambda = 0.001$ and iteration number $I = 10$ with the Kaczmarz reconstruction method in all experiments.

To assess the reconstruction results, utilize the peak signal-to-noise ratio (PSNR) index and structural similarity index measure (SSIM) index between the reconstructed images and the GT. Table 1 and Table 2 provide the PSNR and SSIM results, respectively.

The letters 'D' and 'U' are reconstructed by selecting frequency components from the 'S' phantom letter and observing the reconstruction outcome. Fig 2 shows that when the same number of frequency components are used, the reconstruction outcome is significantly better with the frequency components selected by the GSA algorithm compared to those selected by SNR values. By observing the calculated PSNR and SSIM, it becomes apparent that the reconstruction results obtained through the GSA algorithm are superior to selected based solely on SNR values. We can see that the GSA algorithm reconstruction result for the letter 'S' is much better than the other letters, this outcome is attributed to the utilization of the letter 'S' during the GSA algorithm training phase for frequency components selection. Consequently, it is expected and justifiable that the reconstruction quality for the letter 'S' would surpass that of other letters. In contrast to the SNR reconstruction outcomes with 30% frequency components selection, the visual effect of GSA algorithm reconstruction results seems to have a lot of noise, but the manifest superior quantitative metrics, specifically higher PSNR and SSIM values, which can underscore the efficacy of the GSA algorithm in the reconstruction process.

**Table 1:** The PSNR result of the experiment

| DATA | Train | Test | |
|---|---|---|---|
| Phantom | S | D | U |
| GSA(10%) | 22.7486 | 18.3773 | 18.0351 |
| SNR(10%) | 15.5021 | 14.5742 | 14.0644 |
| SNR(30%) | 18.8787 | 18.2617 | 19.9366 |

**Table 2:** The SSIM result of the experiment

| DATA | Train | Test | |
|---|---|---|---|
| Phantom | S | D | U |
| GSA(10%) | 0.9332 | 0.8588 | 0.7855 |
| SNR(10%) | 0.5238 | 0.5974 | 0.4494 |
| SNR(30%) | 0.8096 | 0.8200 | 0.8698 |

## IV. Conclusions

Compared to selecting frequency components based on SNR values, choosing them with the GSA algorithm can yield superior results in MPI reconstruction with fewer frequency components. This can help optimize the MPI reconstruction process. While the GSA algorithm specializes in selecting frequency component combinations for enhanced reconstruction results, it exhibits prolonged computation times when initialized with a larger number of agents. Exploring methods to expedite the iteration speed stands as a significant question for further investigation.

## Acknowledgments

This work was supported in part by the Focus on Research and Development Plan in Shandong Province 2022CXGC010501 and the National Natural Science Foundation of China under Grant No.82227802.

## Author's statement

Authors state no conflict of interest.

## References

[1] Storath, Martin et al. "Edge Preserving and Noise Reducing Reconstruction for Magnetic Particle Imaging." IEEE transactions on medical imaging vol. 36,1 (2017): 74-85. doi:10.1109/TMI.2016.2593954

[2] Rashedi, Esmat et al. "GSA: A Gravitational Search Algorithm." Inf. Sci. 179 (2009): 2232-2248

[3] Straub, Marcel, and Volkmar Schulz. "Joint Reconstruction of Tracer Distribution and Background in Magnetic Particle Imaging." IEEE transactions on medical imaging vol. 37,5 (2018): 1192-1203. doi:10.1109/TMI.2017.2777878

 

[4] Chen, Xiaojun et al. "The Reconstruction of Magnetic Particle Imaging: Current Approaches Based on the System Matrix." Diagnostics (Basel, Switzerland) vol. 11,5 773. 26 Apr. 2021, doi:10.3390/diagnostics11050773

[5] Knopp, T, and M Hofmann. "Online reconstruction of 3D magnetic particle imaging data." Physics in medicine and biology vol. 61,11 (2016): N257-67. doi:10.1088/0031-9155/61/11/N257

[6] Li, Chaoshun et al. "A chaos embedded GSA-SVM hybrid system for classification." Neural Computing and Applications 26 (2015): 713-721.

 