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Influence of the system matrix on channel leakage artifacts in multi-contrast MPI

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Abstract

Magnetic Particle Imaging (MPI) is a tracer-based medical imaging modality with great potential due to its high sensitivity, high spatiotemporal resolution, and ability to quantify the tracer concentration. Image reconstruction in MPI is an ill-posed problem, which can be addressed by the use of regularization methods. Single- and multi-contrast MPI reconstructions produce different kinds of artifacts. In this work, the multi-contrast MPI channel leakage is introduced and an analysis of the multi-contrast MPI system matrix properties is conducted to understand the source of the multi-contrast MPI channel leakage.

I. Introduction

Magnetic particle imaging (MPI) is a tracer-based medical imaging technique that enables the sensitive and fast imaging of magnetic nanoparticle distributions [1]. More recently, multi-contrast MPI has opened the door for many applications due to its ability to recover distributions from multiple tracer sources simultaneously [2]. Where the single-contrast MPI reconstruction recovers the particle distribution from a measurement using a single system function, the multi-contrast approach uses multiple system functions for reconstruction to separate the signal from different tracers into different channels given a single measurement. Multi-contrast MPI reconstruction results in multi-channel images, where each channel may encode tracers or environment properties, such as temperature [3], viscosity [4], mobility state, material [5], or core size [6].

A common approach to image reconstruction is based on using a system matrix representation. At its core, it aims to solve an ill-posed inverse problem, which in practice leads to noise artifacts in the reconstructed images. The impact of noise in MPI is commonly handled by applying regularization methods, such as Tikhonov. These offer a trade-off between regularization bias and noise artifacts. While noise artifacts typically lack spatial correlation with the reconstructed object and instead manifest as random noisy structures at the edges of the reconstructed image, bias artifacts commonly exhibit spatial correlation with the reconstructed object and manifest as blurring, over-shooting, or over-smoothing. For single-contrast MPI methods to handle these artifacts have been investigated in some work [7–9]. However, an analysis of the multi-contrast MPI reconstruction artifacts and their sources is still missing.

This work aims to briefly investigate multi-contrast MPI reconstruction artifacts, namely the channel leakage, and to analyze properties of the multi-contrast MPI system matrices to understand the source of multi-contrast MPI artifacts.

II. Multi-Contrast MPI Artifacts

Figure 1 shows reconstruction results for single- and multi-contrast MPI from simulated measurements. The

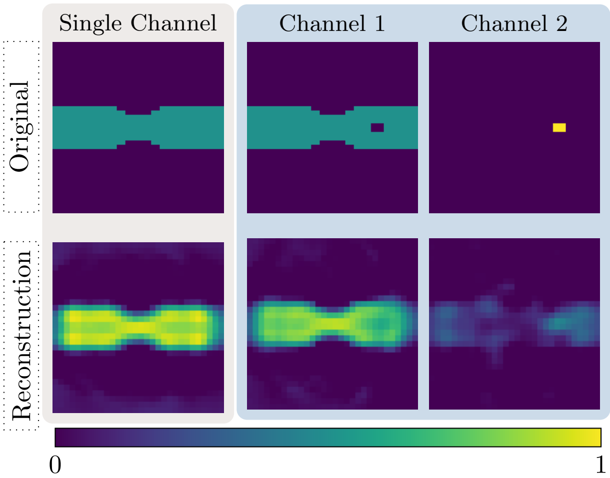

Figure 1: Single- and multi-contrast reconstructions of a vessel/catheter phantom.

first column of the figure shows the reconstruction results of a stenosis phantom in single-contrast MPI. Unstructured artifacts are observed at the top and bottom and an overshooting effect is seen due to the use of Tikhonov regularization. The second and third columns of the figure show the reconstruction results of a multi-contrast MPI simulation consisting of a stenosis and a catheter in the first and second channels, respectively. The first channel of the multi-contrast reconstruction results shows similar artifacts to the ones observed in the single-channel reconstruction, while the second one shows a new type of artifact that is spatially correlated to the object in the first channel. Here, the artifact is so dominant that the marked catheter tip is barely visible. Such artifacts are specific to multi-contrast MPI scenarios and are related to the difficulty of separating the signal between channels, which we refer to as channel leakage.

III. Methods and materials

III.I. Experimental Data

We consider the scenario of catheter tracking in a vessel with stenosis. The vessel phantom is filled with a mobilized tracer and the catheter phantom is marked with an immobilized tracer. The following forward model is used to create the simulated data

$$\underbrace{\begin{pmatrix} \boldsymbol{S}_1 & \boldsymbol{S}_2 \end{pmatrix}}_{\boldsymbol{S}} \underbrace{\begin{pmatrix} \boldsymbol{c}_1 \\ \boldsymbol{c}_2 \end{pmatrix}}_{\boldsymbol{c}} + \boldsymbol{n} = \boldsymbol{u},$$

where $\boldsymbol{S}$ is the system matrix that consists of the mobilized $\boldsymbol{S}_1$ and immobilized $\boldsymbol{S}_2$ system matrices acquired for the two channels concatenated along the spatial axis, $\boldsymbol{u}$ is the measurement vector, and $\boldsymbol{n}$ is the white Gaussian noise vector. The original phantoms used in the simulation are shown in the first row of figure 1, where $\boldsymbol{c}_1$ and $\boldsymbol{c}_1$ are the stenosis and catheter phantoms, respectively.

The mobilized and immobilized perimag-based system matrices $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ were both measured using a preclinical MPI system (Bruker, Ettlingen, Germany). For both system matrices, the drive field amplitudes were 12 $\mathrm{mT}\mu_0^{-1}$ in x- and y- directions and the gradient strength was $G_x = G_y = -1\,\mathrm{Tm}^{-1}\mu_0^{-1}$ and $G_z = 2\,\mathrm{Tm}^{-1}\mu_0^{-1}$ resulting in an effective FOV of 24×24mm. The system matrices were measured on a 30×30 grid on a slightly larger area of 30×30mm. The delta sample had a size of $1 \times 1 \times 1\mathrm{mm}^3$.

III.II. System Matrices Analysis

The objective of analyzing the system matrix is to understand how matrix concatenation affects the properties of the system matrix in multi-contrast MPI. Then, the investigation will focus on the impact of this concatenation on the convergence rate of the widely used iterative Kaczmarz-based reconstruction algorithm [8] and the separation of signals between channels for multi-contrast MPI reconstruction. The following points are considered:

1. Dimensionality: The Kaczmarz-based solver applies to both overdetermined and underdetermined systems, however, its convergence behavior is better for overdetermined systems. Multi-contrast MPI reconstruction requires concatenating the system matrices which implies changing the dimension and the structure of the linear systems. The columns-to-rows ratio (CRR) is introduced to measure the dimensionality change in multi-contrast MPI and is equal to the ratio of the number of columns to the number of rows in the linear system. A square linear system has a CRR of 1. If the CRR is less than 1, the system is overdetermined and if it is greater than 1, the system is underdetermined. The CRR is compared for the mobilized, immobilized, and concatenated system matrices.

2. The condition number: The condition number of a matrix determines the stability and accuracy of solving the inverse problem and influences the convergence behavior of iterative linear solvers, such as the Kaczmarz method. A high condition number leads to slow convergence, while a low condition number results in fast convergence [10]. The condition number is defined as the ratio of the largest and the smallest singular values. The condition number is checked for the mobilized, immobilized, and concatenated system matrices.

3. Similarity of frequency components: When concatenating identical system matrices in multi-contrast MPI, it is noticed that the reconstructed signal gets equally divided on all channels regardless of the phantoms existing in the channels. Thus, it appears that the

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Table 1: Columns to rows ratio for different system matrices.

Columns to Rows ratio	2D
Mobilized System Matrix	1.28
Immobilized System Matrix	1.28
Concatenated System Matrix	2.23

Table 2: The condition number of different system matrices.

The condition number	2D
Mobilized System Matrix	5.1×10^5
Immobilized System Matrix	8.2×10^5
Concatenated System Matrix	3.3×10^6

similarity of the frequency components is a significant factor in causing the channel to leak. To test this hypothesis, a second frequency filtering step is applied to the multi-contrast system matrices based on the similarity of their frequency components. The NRMSD of the corresponding frequency components is used to evaluate their similarity. The concatenated system matrix is frequency-filtered in 3 different ways. First, the commonly used SNR-threshold-based method [11]. Different SNR thresholds are used, namely, 2, 4, 8, and 16. For the second and third methods, the system matrix is first frequency filtered using an SNR threshold of 1, and then as a second frequency filtering step, the frequency components that show higher similarity are selected and then the frequency components that show lower similarity are selected, respectively.

IV. Results

1. Dimensionality: Table 1 displays the CRR of the mobilized, immobilized, and concatenated system matrices. The table shows that the CRR of the concatenated system matrix is higher than the CRR of the mobilized, and immobilized system matrices. Thus, concatenating the system matrices for reconstruction in multi-contrast MPI significantly increases the under-determination of the joint linear system.

2. The condition number: Table 2 displays the condition number of the mobilized, immobilized, and concatenated system matrices. The table shows that the condition number of the joint system matrix is higher than it is for the single ones. This indicates that concatenating the system matrices in multi-contrast MPI implies increasing the condition number of the resulting joint system matrix.

3. Similarity of frequency components: Figure 2 shows the reconstruction results of the second channel, showing the catheter phantom, using system matrices with different ways of frequency filtering. The parameter K represents the number of selected frequency components. The first row displays reconstruction results using SNR-based frequency-filtered system matrices. The second row shows reconstruction results using the system matrices with high-similarity-frequency selection, and the third row uses the system matrices with low-similarity-frequency selection.

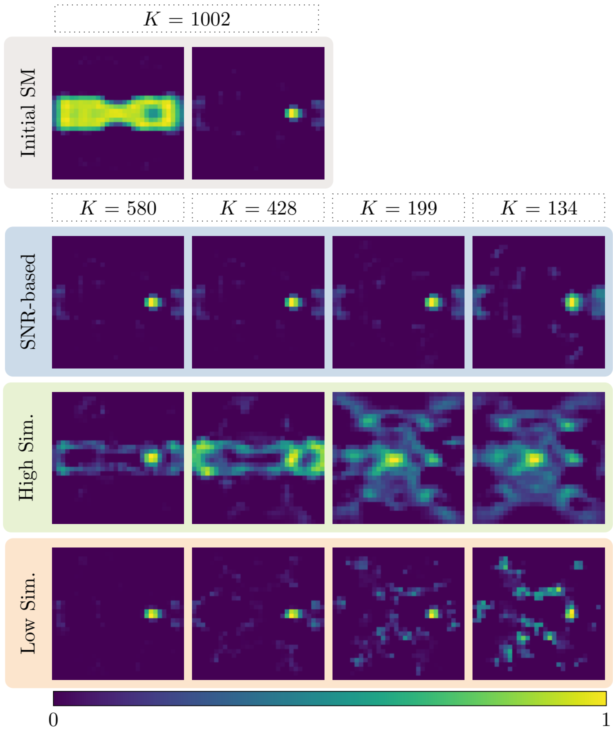

Figure 2: The reconstruction results of the second channel using system matrices with different ways of frequency filtering.

The reconstruction results obtained with the system matrices including the high-similarity-frequency components have the highest amount of leakage in the second channel. Reconstruction results with the low-similarity-frequency components show slightly less leakage into the second channel compared to the reconstructions with standard system matrices. However, when using a small number of the frequency components for reconstruction, one notices more noise artifacts in the low-similarity-frequency selection than in the standard one.

V. Conclusions & Discussion

Concatenating the system matrices for reconstruction in multi-contrast MPI significantly increases the under-determination of the joint system, which usually slows the convergence of the reconstruction. It also implies increasing the condition number of the resulting joint system matrix, which affects the convergence of the iterative solver, i.e. more iterations are required to reach

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satisfactory reconstruction results and less channel leakage using the Kaczmarz-based reconstruction algorithm. From the reconstruction results in Figure 2, one can conclude that the higher the similarity of the corresponding frequency components in the concatenated system matrices, the higher the channel leakage. Thus, a second frequency filtering step based on the similarity of the corresponding frequency components could help reduce the channel leakage artifacts in multi-contrast MPI reconstructions.

References

[1] B. Gleich and J. Weizenecker. Tomographic imaging using the nonlinear response of magnetic particles. *Nature*, 435(7046):1214–1217, 2005, doi:10.1038/nature03808.

[2] J. Rahmer, A. Halkola, B. Gleich, I. Schmale, and J. Borgert. First experimental evidence of the feasibility of multi-color magnetic particle imaging. *Physics in Medicine & Biology*, 60(5):1775, 2015.

[3] C. Stehning, B. Gleich, and J. Rahmer. Simultaneous magnetic particle imaging (mpi) and temperature mapping using multi-color mpi. *Int J Mag Part Imag*, 2, 2016.

[4] M. Möddel, C. Meins, J. Dieckhoff, and T. Knopp. Viscosity quantification using multi-contrast magnetic particle imaging. *NEW J PHYS*, 20:083001, 2018.

[5] J. Rahmer, A. Halkola, B. Gleich, I. Schmale, and J. Borgert. First experimental evidence of the feasibility of multi-color magnetic particle imaging. *Physics in medicine and biology*, 60(5):1775–1791, 2015, doi:https://doi.org/10.1088/0031-9155/60/5/1775.

[6] C. Shasha, E. Teeman, K. Krishnan, P. Szwargulski, T. Knopp, and M. Möddel. Discriminating nanoparticle core size using multi-contrast MPI. *Physics in Medicine & Biology*, 64(7):074001, 2019, doi:10.1088/1361-6560/ab0fc9.

[7] A. Weber, F. Werner, J. Weizenecker, T. M. Buzug, and T. Knopp. Artifact free reconstruction with the system matrix approach by overscanning the field-free-point trajectory in magnetic particle imaging. *Physics in Medicine & Biology*, 61(2):475, 2015, doi:10.1088/0031-9155/61/2/475.

[8] T. Knopp, J. Rahmer, T. F. Sattel, S. Biederer, J. Weizenecker, B. Gleich, J. Borgert, and T. M. Buzug. Weighted iterative reconstruction for magnetic particle imaging. *Physics in medicine and biology*, 55(6):1577–1589, 2010, doi:10.1088/0031-9155/55/6/003.

[9] L. Nawwas, C. Brandt, P. Szwargulski, T. Knopp, and M. Möddel. Reduction of bias for sparsity promoting regularization in mpi. *IJMPI*, 7(2):1–3, 2021, doi:https://doi.org/10.18416/IJMPI.2021.2112002.

[10] A. Pyzara, B. Bylina, and J. Bylina, The influence of a matrix condition number on iterative methods' convergence, 459–464, 2011.

[11] T. Knopp and T. M. Buzug, Magnetic Particle Imaging: An Introduction to Imaging Principles and Scanner Instrumentation. Berlin/Heidelberg: Springer, 2012, doi:10.1007/978-3-642-04199-0.

10.18416/ijmpi.2024.2403033