

Proceedings Article

Model-Based Reconstruction in MPI accounting for Field Imperfections

Sarah Reiss ^{a,b,*} · Florian Thieben ^{a,b} · Jonas Faltinath ^{a,b} · Tobias Knopp ^{a,b,c} ·
Marija Boberg ^{a,b}

^aSection for Biomedical Imaging, University Medical Center Hamburg-Eppendorf, Hamburg, Germany

^bInstitute for Biomedical Imaging, Hamburg University of Technology, Hamburg, Germany

^cFraunhofer Research Institution for Individualized and Cell-based Medical Engineering IMTE, Lübeck, Germany

*Corresponding author, email: sarah.reiss@tuhh.de

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Abstract

To date a system matrix has to be obtained through a tedious calibration measurement when employing a system matrix-based reconstruction in magnetic particle imaging. This problem can be effectively addressed by model-based reconstruction, which takes into account both particle and scanner parameters. In this study, we focus on the scanner parameters and in particular on the fact that the fields of experimental systems are imperfect. For experimental Lissajous-type data we show that the modeling error can be substantially reduced by about 18 % by incorporating field imperfections in both the transmit and receive coils.

I. Introduction

In order to reconstruct an image in magnetic particle imaging (MPI), either the x -space or the system matrix (SM)-based reconstruction is used. In the latter case, an SM is employed that contains information regarding both the particle parameters and the scanner parameters [1, 2]. The current state of the art for obtaining an SM is a tedious calibration measurement, which has the advantage of including all relevant information. However, the calibration process is time-consuming and has to be repeated whenever there is a change of the used particles, the scanning sequence or the scanner hardware. One potential solution to this issue is to model the SM. To date, the majority of research has focused on modeling the dynamics of the particles, under the assumption of ideal fields, yielding promising outcomes [3]. A further issue are the scanner parameters [1]. In some MPI scanners, the fields of the transmit and receive coils are often spatially non-ideal, especially off-center. While

the simulations of Maass *et al.* [3] are restricted to an area with almost ideal fields, Bringout *et al.* [4] demonstrated the influence of non-ideal magnetic fields for a field-free-line imaging sequence on simulated data.

The purpose of this work is to account for field imperfections when modeling a 2D Lissajous-type SM. To this end, we use data measured with a human head scanner [5] and show that the modeling error can be significantly reduced by incorporating measured non-ideal fields.

II. Methods and materials

The continuous system function $\hat{s}_{l,k} : \mathbb{R}^3 \rightarrow \mathbb{C}$ can be defined for each spatial position $\mathbf{r} \in \mathbb{R}^3$, receive channel $l \in \mathbb{N}$, and frequency index $k \in \mathbb{N}_0$ as

$$\hat{s}_{l,k}(\mathbf{r}) = -\hat{a}_{l,k} \mathbf{p}_l(\mathbf{r}) \frac{\mu_0}{T} \int_0^T \frac{\partial}{\partial t} \bar{\mathbf{m}}(\mathbf{H}(\mathbf{r}, t)) e^{-\frac{2\pi i k t}{T}} dt \quad (1)$$

where μ_0 denotes the permeability of vacuum and T the period of the trajectory. The system function further

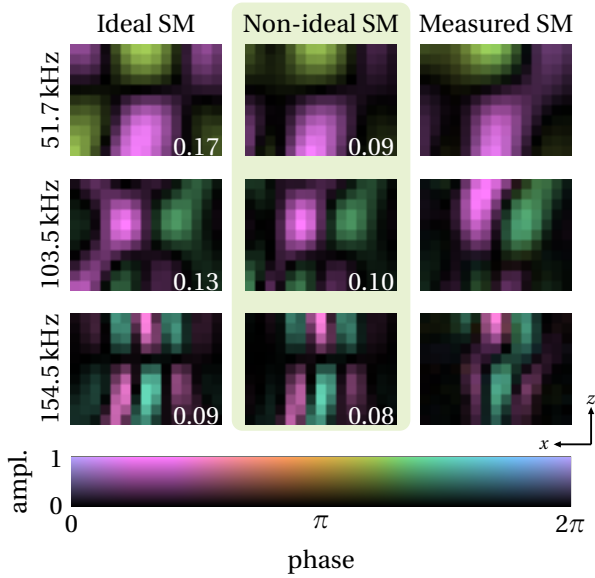


Figure 1: Comparison of different SMs for three different frequencies. For each frequency the corresponding ideal, non-ideal, and measured SM are shown at the central xz -plane for the x -receive channel. The errors to the measured SM at the depicted frequencies are shown in the lower right corner.

consists of the analog filter kernel $\hat{a}_{l,k} \in \mathbb{C}$ of the receive chain, the sensitivity-field profile $\mathbf{p}_l : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ of the l -th receive coil, and the time derivative of the mean magnetic moment $\dot{\mathbf{m}} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, which depends on the applied magnetic fields $\mathbf{H} : \mathbb{R}^3 \times [0, T] \rightarrow \mathbb{R}^3$.

Non-ideal fields deform a 2D planar Lissajous field-free-point trajectory in all three spatial directions. Hence, we simulated the SM (*non-ideal SM*) on a 3D grid ($140 \times 110 \times 100$ mm) for the MPI scanner described in [5] with a static selection field. The remaining scanner parameters were selected in accordance with the aforementioned MPI scanner. The fields (\mathbf{p}_l and \mathbf{H}) were measured and calculated using spherical harmonics as described in [2, 6, 7]. The analog filter kernel ($\hat{a}_{l,k}$) was measured and estimated as described in [2, 8].

In order to simulate the mean magnetic moment ($\dot{\mathbf{m}}$), the equilibrium model that accounts for particle anisotropy [3] with the particle parameters $D = 23$ nm and $K^{\text{anis}} = 500$ J m⁻³ was employed. The software package *MNPDynamics.jl*¹ was used for the computation.

The same SM was modeled assuming ideal fields (*ideal SM*). For quantification we used the normalized root mean square error to the measured SM [5].

III. Results and discussion

Figure 1 shows exemplary images of the non-ideal SM in comparison to the ideal and measured one for three

frequencies 51.7 kHz (top), 103.5 kHz (middle), and 154.5 kHz (bottom). The depicted slice is the central xz -plane. Notably, the non-ideal SM is in closer alignment with the measured one, particularly in the outer regions. Here the magnetic fields are less ideal and therefore the ideal SM has a higher discrepancy. Considering the significant frequencies of the x -receive channel, the mean error for the ideal SM is 0.129 while it is 0.106 for the non-ideal SM.

IV. Conclusion

We found that using measured magnetic fields to simulate the SM resulted in an improvement of about 18 % for the specified MPI scanner [5]. This is an important step towards model-based image reconstruction for scanners with medium to large field imperfections and has the potential to significantly reduce the calibration effort in MPI.

Author's statement

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¹<https://github.com/MagneticParticleImaging/MNPDynamics.jl>