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Neural implicit representations for grid-agnostic MPI reconstructions

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Abstract

Magnetic particle imaging (MPI) reconstructs the spatial distribution of magnetic nanoparticles on a fixed grid, the resolution of which is limited by the noise present in the system. This paper addresses the reconstruction problem while integrating single-image super-resolution for concentration maps. We introduce Neural Implicit Representations (NIR) as an image prior, enabling arbitrary grid size sampling after training. Experimental results using a spiral phantom measurement reveal that NIR-based reconstruction maintains image sharpness across diverse grid sizes, surpassing the two-stage Kaczmarz- ℓ_2 reconstruction followed by bicubic up-sampling in preserving fine structural details. This technique has a potential for high-resolution MPI imaging without relying on extensive datasets.

I. Introduction

Magnetic particle imaging (MPI) obtains spatial distribution of magnetic nanoparticles excited with dynamic magnetic field. The reconstruction problem is a linear ill-posed inverse problem defined by the system matrix (SM) and commonly solved using regularization in combination with iterative solvers. Recently, neural networks proved to have regularization effect in MPI reconstruction [1], an approach known as Deep Image Prior (DIP).

The SM is usually measured using a small delta sample on a fixed grid. The size of the sample is selected based on the trade-off between SNR and resolution, the latter of which is determined by the properties of the nanoparticles and the magnetic field gradient. The resolution of the resulting reconstructions can be improved either by pre-processing SM [2] or post-reconstruction image processing [3].

While the two problems can be addressed independently, joint formulations are possible [4]. In this paper, we employ Neural Implicit Representations (NIRs) [5] as

a prior in the reconstruction problem which later allows to improve the resolution of MPI reconstructions. Unlike DIP which takes a fixed noise sample as an input, NIRs map grid coordinates to a desired property and, once optimized, allow arbitrary grid-sizes to be sampled.

II. Methods and materials

In this work, NIR is a fully-connected neural network with periodic activation functions [6] mapping a vector of normalized coordinates \mathbf{p} to real-valued concentration values. In this setup, the reconstruction is done by training the network

$$\operatorname{argmin}_{\theta} \| \mathbf{S} D \varphi_{\theta}(\mathbf{p}) - \mathbf{u} \|_2, \quad (1)$$

i.e. optimization of the network weights, where $\mathbf{S} \in \mathbb{C}^{M \times N}$ is the SM, $\mathbf{u} \in \mathbb{C}^M$ are the measurements, D is the down-sampling operator and $\varphi_{\theta}(\mathbf{p})$ is the NIR parametrized by θ . For each iteration, we randomly sample coordinates corresponding to one of the finer grids.

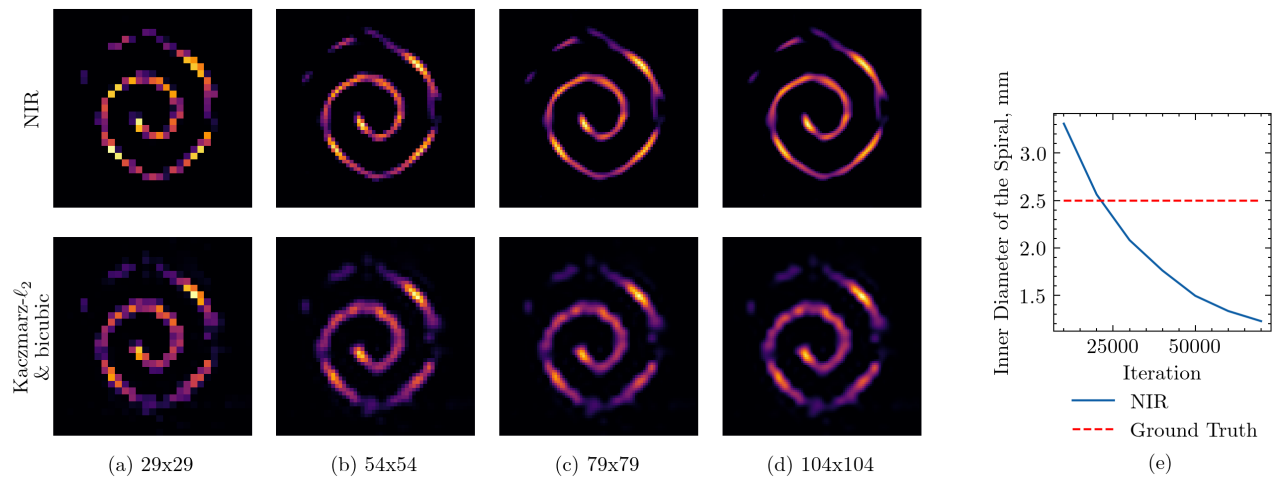


Figure 1: (a)-(d) Reconstructions of the spiral phantom sampled at different grid sizes: 29^2 (native), 54^2 , 79^2 , 104^2 ; (e) Dependence of the spiral's inner diameter on the number of training iterations.

E.g., having $N = a \times b$, the grids represent $sa \times sb$, where $s = 1, \dots, 5$. We then apply average pooling with the corresponding kernel to align dimensions with the SM.

We conduct an experiment using a 2D measurement of a spiral phantom. The SM has $29 \times 29 \text{ mm}^2$ FOV with native grid size of 29×29 . We subtract mean background signal and select frequency in the range between 20 and 380 kHz with SNR larger than 1.5. The data is normalized according to the ℓ_2 norm of the SM rows. An adopted version of Kaczmarz method with ℓ_2 regularization is run for 150 iterations with $\lambda = 0.5$ to reconstruct on the native 29×29 grid. We use bilinear interpolation as a baseline method for resampling. The NIR is optimized using Adam with learning rate 10^{-4} for 70K iterations. The two methods are compared visually using finer grids.

III. Results

Figure 1(a-d) presents a comparison between NIR-based reconstructions and interpolated Kaczmarz- ℓ_2 reconstructions across various grid sizes. Unlike bicubic up-sampling, the proposed method preserves the sharpness of the reconstructions at all grid sizes, maintaining the structural integrity of curved features.

Figure 1(e) highlights a notable characteristic of the proposed method: as the number of training iterations increases, the spiral becomes progressively thinner. Similar to DIP, the proposed method requires early stopping in the training process to prevent overfitting.

IV. Discussion and Conclusion

We proposed a novel reconstruction method based on NIR that both regularizes the inverse problem and en-

ables sampling at arbitrary grid sizes. As training is conducted per measurement, the approach does not rely on large datasets. However, the method's primary drawback lies in the number of hyper-parameters that require tuning for each new measurement. Future work should focus on evaluating the stability of the reconstruction method across diverse measurements.

Author's statement

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