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Experimental Study on Efficient Field Measurement using Ellipsoidal Harmonics

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Abstract

This work demonstrates the application of ellipsoidal harmonic expansions for magnetic field representation in Magnetic Particle Imaging (MPI) using measured data. While previous studies focused on simulated fields, we validate the method with field data acquired from a selection field generator of an MPI head imaging system using an ellipsoidal t-design. The approach is validated by comparing the harmonic expansion results to dense principal axis measurements, confirming accurate field characterization. This study highlights the potential of ellipsoidal harmonics to improve the speed and accuracy of magnetic field measurements and representation in MPI.

I. Introduction

Understanding magnetic fields in Magnetic Particle Imaging (MPI) is essential for optimizing system performance and accuracy. Field inhomogeneities or distortions can cause inaccurate spatial representation of nanoparticles, affecting diagnostic outcomes or industrial processes. Accurate field knowledge allows for improved system design, leading to higher image quality and more precise nanoparticle localization.

Measuring magnetic fields on a dense three-dimensional grid is time-consuming and inefficient. To overcome this, more efficient measurement techniques have been developed enabling the magnetic field to be effectively represented as a series expansion of harmonic functions [1, 2]. To determine the coefficients of this expansion, only a limited number of measurement points, arranged according to t-designs, are required. These points are typically distributed on a spherical surface. Recently, the use of Ellipsoidal Harmonic Expansions (EHE) was introduced. They offer two more degrees of freedom enabling a much more flexible and efficient description of magnetic fields in MPI as scanner bores usually have cylindrical shape [3].

While ellipsoidal harmonics have previously been applied to simulated data, this study extends their application to experimentally acquired data. We demonstrate the effectiveness of this approach by collecting field data from the gradient field of our institute's head imaging system [4] using an ellipsoidal t-design. To assess the accuracy and reliability of the method we compare the ellipsoidal harmonic representation with dense measurements taken along the principal axes.

II. Methods and Materials

Source-free magnetic fields $\boldsymbol{B} : \mathbb{R}^3 \supset \Omega \to \mathbb{R}^3$ can be represented by a polynomial of maximum degree $N \in \mathbb{N}$. Thus, using an EHE within a reference ellipsoid defined by it's semi-axes $(a_1, a_2, a_3) \in \mathbb{R}^3$, the field can be expressed as:

$$\boldsymbol{B}(\rho, \mu, \nu) = \sum_{n=0}^{N} \sum_{m=1}^{2n+1} \boldsymbol{A}_{a_1}^{n,m} \mathbb{E}_n^m(\rho, \mu, \nu). \quad (1)$$

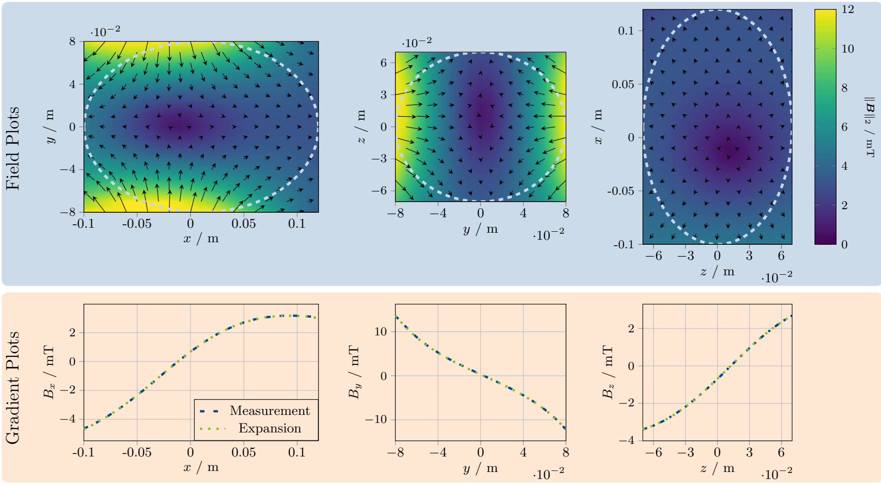

Figure 1: Top row: sectional views of the magnetic field resulting from the ellipsoidal harmonic expansion. Bottom row: comparison of the field values with the test measurements along the principle axis.

Here (ρ, μ, ν) are ellipsoidal coordinates, $\mathbb{E}_n^m : \mathbb{R} \to \mathbb{R}$ are inner ellipsoidal harmonics of degree $n \in \mathbb{N}$ and order $m \in \{1, ..., 2n+1\}$ and $A_{a_1}^{n,m} \in \mathbb{R}$ denote the according coefficients. To compute these coefficients, surface integrals over the reference ellipsoid need to be solved, for which we employ an ellipsoidal t-design as a quadrature. For further details on the theory of ellipsoidal harmonics, ellipsoidal t-designs and how to use them to describe magnetic fields, we refer to [3, 5].

A reference ellipsoid with$\boldsymbol{a} = (0.11, 0.08, 0.07)$ m centered at $(0.01, 0, 0)$ m was chosen to cover the scanner bore of the imaging system described in [4]. An ellipsoidal t-design with $t = 14$ was used, defining 114 measurement points on the surface of the reference ellipsoid. Measurements used a three-axis Hall-effect sensor on a robot connected to a 3-channel Gaussmeter (M460, Lake Shore Cryotronics). For verification, measurements were taken at 55 control points (Ω_{ctrl}) with equidistant intervals of 0.01 m on the three main axes: 23 in x-direction ($y = z = 0$ m), 17 in y-direction ($x = 0.01$ m, $z = 0$ m) and 15 in z-direction ($x = 0.01$ m, $y = 0$ m). A quantitative evaluation is done by calculating the mean relative error normalized by the mean field.

III. Results

In Figure 1 sectional views of the magnetic field obtained from the EHE are shown for each central plane. The characteristics of the MPI selection field are well represented and the dominant y-gradient is clearly visible. White dashed lines show the reference ellipsoid on which the points of the t-design are located. A comparison of the gradients on the control points is shown along the principal axes in the second row. No visual differences are apparent. Quantitative analysis confirms a high accuracy with a mean error of $\xi = 0.78\,\%$.

IV. Discussion and Conclusion

The results clearly demonstrate that EHE are a powerful tool for measuring and representing magnetic fields in MPI. While previous work has only used simulated field data, we have shown in this study that the method can also be applied to measured data. The greater flexibility of the geometric shape of an ellipsoid permits superior coverage of typical MPI scanner bores in comparison to the well-established spherical harmonic expansions. Potential measurement inaccuracies did not affect the stability of the method in our case as the mean error below 1 % confirms. However, a sophisticated error analysis remains for future work.

Author's statement

Conflict of interest: Authors state no conflict of interest.

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