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30-fold acceleration using sinogram-based system calibration for field free line MPI

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Abstract
We propose a field-based calibration directly in the sinogram domain for a 3D field free line (FFL) dynamic sinusoidal trajectory. This approach involves performing calibration measurements with alternating magnetic field offsets for each offset and z-position in sinogram domain (covering a total of $N_{xy} \times N_z$ positions on a 2D grid), rather than calibration measurements for each MPI image voxel in 3D space (e.g., for $N_{xy} \times N_{xy} \times N_z$ voxels). Using the sinogram-based calibration data, we synthesize the 3D system matrix (SM) in the image domain by exploiting the shift and rotation invariance of MPI systems. The proposed 2D sinogram-based calibration method for the 3D FFL trajectory achieved a 30-fold reduction in system calibration time due to reduced dimensionality, with less than 0.5% normalized RMS-error compared to 3D image-based calibration.

I. Introduction

In field free line (FFL) based Magnetic Particle Imaging (MPI), a 3D field-of-view (FOV) can be imaged by, e.g., rotating and translating the FFL dynamically on a plane and by using solely a 1D excitation orthogonal to that plane. This can be achieved, for example, with a dynamic sinusoidal trajectory with duration T_{acq}, described by $r(t) = \frac{\text{FOV}_{xy}}{2} \sin(2\pi f_{\text{tra}} t)$ and $\theta(t) = 2\pi f_{\text{rot}} t$, where r and θ are the FFL-offset with respect to the origin $(x = y = 0)$ and FFL-angle, respectively, FOV_{xy} is the FOV in the xy-plane, and f_{tra} and f_{rot} are the translational and rotational frequencies of the FFL, respectively, which must be set appropriately to ensure periodicity and achieve optimal sinogram coverage (Fig. 1).

In this study, a new field-based calibration method (similar to [1]) is proposed for dynamic FFL trajectories but by performing 2D calibration measurements in sinogram domain with alternating (i.e., ac) magnetic field offsets, specifically tailored for the 3D FFL sinusoidal trajectory. Using the sinogram-based calibration data, the

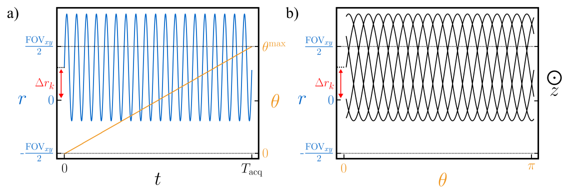

Figure 1: (a) 3D FFL dynamic sinusoidal trajectory with 1D excitation in the z-direction (not shown), (b) represented in sinogram domain. Dashed lines at $\pm\frac{\text{FOV}_{xy}}{2}$ indicate boundaries of the trajectory without trajectory offsets Δr_k.

3D SM in the image domain was synthesized by leveraging the shift and rotation invariance of the MPI system [2].

II. Methods

The sinogram-based system calibration sequence for the 3D FFL sinusoidal trajectory is defined by $r_k(t) = r(t) + \Delta r_k$, where

$\Delta r_k = -\frac{\mathrm{FOV}_{xy} - d_{xy}}{2} + k d_{xy},\ k \in \{0, 1, \cdots, N_{xy} - 1\}$ are trajectory offsets at k^{th} repetition, and d_{xy} and N_{xy} are the voxel size and the number of voxels in the xy-directions, respectively. The calibration measurements with N_{xy} trajectory offsets are repeated for each voxel of the FOV_z with a calibration probe in the center line of MPI system bore along the z-direction. Assuming 100% acquisition duty cycle, the total measurement time is therefore $N_{xy} \times N_z \times T_{\mathrm{acq}}$, where N_z is the number of voxels in the z-direction. The resulting sinogram-based calibration data can be represented as $\mathbf{A}_{\mathrm{sino}}(t, r, z)$ for positions (r, z).

II.I. Synthesis of the 3D system matrix

Assuming that the magnetic fields are shift and rotation invariant, and the FFL has a straight- and infinite-line formation, the MPI signal $s(t, x, y, z)$, can be formulated in terms of $\mathbf{A}_{\mathrm{sino}}(t, r, z)$:

$$s(t, x, y, z) = \mathscr{R}_{\theta(t)}\{\Psi\{\mathbf{A}_{\mathrm{sino}}(t, r, z)\}\}, \tag{1}$$

where $x = r\cos\theta$, $y = r\sin\theta$, Ψ is the operator replicating $\mathbf{A}_{\mathrm{sino}}$ along its r-dimension, resulting in a matrix of size $T \times N_{xy} \times N_{xy} \times N_z$, $\mathscr{R}_{\theta(t)}$ is the rotation operator by the angle $\theta(t)$ around the z-axis. Using this relation, the 3D SM in image domain, $\mathbf{A}$, can be synthesized by concatenating the resulting rotated data per time and calculating its Fourier transform as follows:

$$\mathbf{A}_{\mathrm{time}}(x, y, z) = \begin{bmatrix} \mathscr{R}_{\theta(1)}\{\Psi\{\mathbf{A}_{\mathrm{sino}}(t(1), r, z)\}\} \\ \mathscr{R}_{\theta(2)}\{\Psi\{\mathbf{A}_{\mathrm{sino}}(t(2), r, z)\}\} \\ \vdots \\ \mathscr{R}_{\theta(T)}\{\Psi\{\mathbf{A}_{\mathrm{sino}}(t(T), r, z)\}\} \end{bmatrix} \tag{2}$$

$\mathbf{A} = \mathscr{F}\{\mathbf{A}_{\mathrm{time}}\}$, where T is the number of sample points per measurement, and $\mathscr{F}$ is the Fourier transform operator along the time dimension.

II.II. In silico evaluation

The proposed sinogram-based calibration method was compared against the conventional image-based calibration in simulations, using the following parameters: G=4 T/m, f_{df}=25 kHz, A_{df}=30 mT, T_{acq}=100 ms, FOV_{xy}=60 mm, FOV_z=20 mm with overscanning, N_{xy}=30, N_z=10, f_{tra}=180 Hz, and f_{rot}=50 Hz. Gaussian noise was added to attain 50 dB SNR. The normalized root-mean-squared error (nRMSE) between the synthesized SM $\mathbf{A}$ and the conventional SM $\mathbf{A}_{\mathrm{conv}}$ was calculated using the formula

$$E(f) = \frac{\sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(|\mathbf{A}(f, n)| - |\mathbf{A}_{\mathrm{conv}}(f, n)|\right)^2}}{\max(|\mathbf{A}_{\mathrm{conv}}(f)|) - \min(|\mathbf{A}_{\mathrm{conv}}(f)|)}, \tag{3}$$

where $N = N_{xy} \times N_{xy} \times N_z$ is the number of voxels.

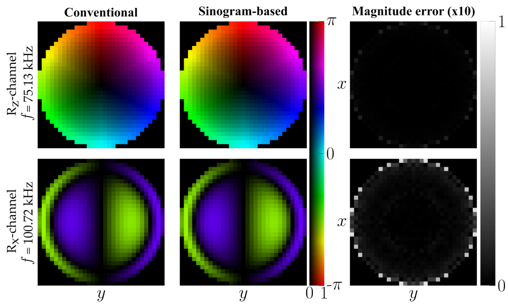

Figure 2: Normalized complex frequency components of SMs (z=0 plane). $f = 3f_{\mathrm{df}} + f_{\mathrm{tra}} - f_{\mathrm{rot}} = 75.13$ kHz, R_z-channel (upper row) and $f = 4f_{\mathrm{df}} + 4f_{\mathrm{tra}} = 100.72$ kHz, R_x-channel (lower row), with 10x amplified error maps.

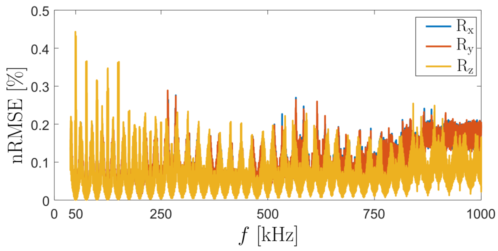

Figure 3: nRMSE per frequency for each receiver channels

III. Results and discussion

The synthesized 3D SM $\mathbf{A}$ bears close resemblance to the conventional SM $\mathbf{A}_{\mathrm{conv}}$, despite N_{xy}=30-fold shorter total measurement time. Exemplary frequency components (75.13 kHz and 100.72 kHz) are shown in Fig. 2 with 10x amplified magnitude error maps, where the errors arise from numerical inaccuracies when calculating $\mathbf{A}_{\mathrm{time}}$ using Equation 2. The maximum nRMSE was less than 0.5% as shown in Fig. 3. The effect of replication of the sinogram-based calibration data N_{xy}-times should be further investigated. Additionally, the maximum FFL-offset r, may be very constrained due to power limitations.

IV. Conclusion

2D sinogram-based robot-free calibration enables 30-fold faster system calibration for the 3D FFL dynamic sinusoidal trajectory without significant error (<0.5%) compared to the 3D image-based calibration.

Author's statement

Conflict of interest: Authors currently work for Bruker BioSpin GmbH & Co. KG.

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