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Efficient Iterative Reconstruction for an MPI Equilibrium Model with Anisotropy

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Abstract
Image reconstruction in Magnetic Particle Imaging (MPI) typically requires a system matrix, obtained through a time-consuming calibration process. To bypass this, various model-based approaches have been explored. Recent work demonstrated successful reconstruction by adapting a Chebyshev approach with Tikhonov-regularized least squares (LS) under an equilibrium model with anisotropy. In this study, we introduce an efficient evaluation of the forward and adjoint operators for the anisotropy model, enabling the use of iterative solvers and alternative regularization methods for image reconstruction.

I. Introduction

Recently, an equilibrium model with anisotropy has been proposed, which allows for accurate modeling of Lissajous type MPI system matrices [1]. The reconstruction problem under this model can be expressed for a single receive coil as a two-fold problem [2], in which we have to solve the following integral equations:

$$\hat{u}_k = a_k \int_{\mathbb{R}^3} \tilde{c}(\mathbf{y}) P_k^{(2)}(\mathbf{y})\, \mathrm{d}\mathbf{y}, \tag{1a}$$

$$\tilde{c}(\mathbf{y}) = \int_{\mathbb{R}^3} c(\mathbf{x}) \mathbf{p}^T \boldsymbol{K}\big(\mathbf{x}, \mathbf{x}-\mathbf{y}\big)\, \mathrm{d}\mathbf{x} =: Ac(\mathbf{y}), \tag{1b}$$

where $\mathbf{p} \in \mathbb{R}^3$ is the sensitivity vector, $a_k \in \mathbb{C}$ are frequency-dependent coefficients, $P_k^{(2)}: \mathbb{R}^3 \to \mathbb{C}$ are functions related to a tensor product of Chebyshev polynomials and $\boldsymbol{K}: \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}^3$ is the anisotropy model spatially-variant kernel.

An efficient approximation to $\tilde{c}$ is described in [2]. Thus, the remaining challenge is efficiently solving for the concentration c, given $\tilde{c}$. Iterative solvers like FISTA or the Richardson-Lucy (RL) algorithm can be used for this purpose, requiring evaluation of the operator (1b) and its adjoint. However, under the anisotropy model the spatially-variant kernel prevents using fast implementations such as the FFT, making the process slow. To address this, [3] proposes a rank-p Karhunen-Loéve (KL) approximation of the kernel:

$$\tilde{K}(\mathbf{u}, \mathbf{x}) = \sum_{k=1}^{p} z_k(\mathbf{u}) e_k(\mathbf{x}), \tag{2}$$

allowing the operator (1b) and its adjoint to be approximated as a sum of convolutions and cross-correlations respectively, with shift-invariant kernels:

$$Ac(\mathbf{x}) \approx \sum_{k=1}^{p} \int_{\mathbb{R}^3} c(\mathbf{u}) z_k(\mathbf{u}) e_k(\mathbf{x}\text{-}\mathbf{u})\, \mathrm{d}\mathbf{u}, \tag{3a}$$

$$A^* b(\mathbf{u}) \approx \sum_{k=1}^{p} \int_{\mathbb{R}^3} b(\mathbf{x}) z_k(\mathbf{u}) e_k(\mathbf{x}-\mathbf{u})\, \mathrm{d}\mathbf{x}, \tag{3b}$$

which enable the use of the FFT to approximate (1b).

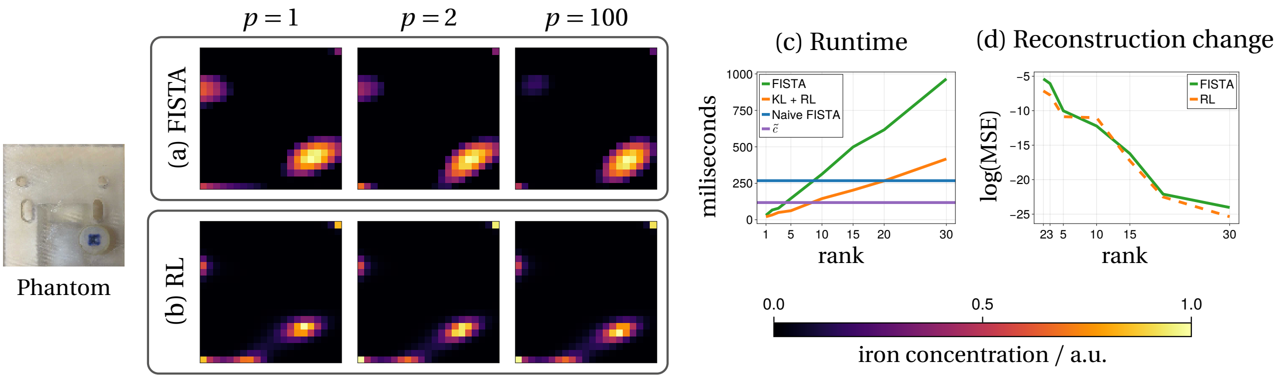

Figure 1: Results of the reconstruction, on the same color scale for comparison. (a) Reconstruction via FISTA with L^1 regularization. (b) Reconstruction via RL. (c) Runtime of the reconstructions in ms with respect to the rank. In blue is the runtime of FISTA with the convolution matrix of the Direct Chebyshev Reconstruction (DCR) in [2]. In purple is the computation time for $\tilde{c}$. (d) Change in consecutive reconstructions with respect to the rank, measured via the MSE.

II. Methods and Materials

To test the reconstruction, we use the "UT" anisotropy kernel type [4] in the first receive channel only. The reconstructions shown in Figure 1 were created using measurement data of a single point phantom. Details on the data acquisition can be found in [1]. For reconstruction with FISTA, the kernel is generated on a $41x41x21x21$ grid to minimize artifacts. In contrast, RL requires the same discretization for both variables, so the kernel is generated on a $21x21x21x21$ grid. The imaginary part of $\tilde{c}$ is removed prior to reconstruction. Negative values of $\tilde{c}$ are clipped to zero for RL, as it only accepts positive measurements. FISTA reconstruction produces negative artifacts, which are clipped to zero as well. Both methods support various regularization techniques, but only L^1 regularization is used here in FISTA, with regularization parameter $\lambda = 0.15$ and 70 iterations. The reconstruction with RL is performed in 30 iterations. All reconstructions are normalized to $[0, 1]$ for a fair comparison with heatmaps.

III. Results and discussion

As shown in Figure 1, a minimum rank one is sufficient to provide a functional reconstruction in both FISTA and RL. Increasing the rank slightly enhances the quality of the reconstruction and reduces the artifacts for FISTA, but increases them for RL. Further increases in rank have minimal effect, as illustrated quantitatively in plot (d) in Figure 1. That is because the eigenvalues corresponding to the first few summands are significantly larger than the rest and also account for nearly the entire sum of eigenvalues (e.g., $\lambda_1 = 1216.36$, representing the 99.03% of the total sum), hence the remaining terms in the approximation are virtually negligible.

As shown in Figure 1, the runtime is linear on the rank of the KL approximation used, and the reconstruction with low ranks is faster than a naive iterative reconstruction using FISTA with the DCR convolution matrix, in the same number of iterations and regularization.

IV. Discussion and Conclusion

A low-rank approximation of the anisotropy kernel is accurate enough to perform image reconstruction.

This approach provides an efficient evaluation of (1b), and enables efficient use of iterative solvers for image reconstruction under the equilibrium model with anisotropy, offering flexibility in terms of speed, image quality, and regularization. Further developments can be made to perform a joint reconstruction using the measurements from both receive coils.

Author's statement

Conflict of interest: Authors state no conflict of interest. We acknowledge Marco Maass and Christine Droigk for sharing the DCR anisotropy reconstruction code.

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