

Proceedings Article

Deconvolution of direct reconstructions in 3D

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Abstract

A while ago a direct reconstruction method for multi-dimensional MPI was proposed, which is based on weighting frequency components of the measured voltage signals with Chebychev polynomials of second kind. The method works fast but leads to reconstructions of convolved spatial distributions of magnetic nanoparticles. In a previous work we were able to show that using a neural network model to deconvolve these reconstructions leads to high-quality images in the two-dimensional case. In this work, we take this approach one step further and demonstrate that this also applies to three-dimensional data. Therefore, in this work, we apply a neural network model on a simulated data set consisting of three-dimensional volumes containing blood vessel like structures. We show that the proposed network produces high-quality deconvolution results and outperforms conventional methods on the data set.

I. Introduction

The direct Chebychev reconstruction method (DCR) from [1] relies on the system function in the Langevin model of paramagnetism with Lissajous-type excitation patterns. As it was shown before, in field-free point (FFP) MPI the convolved distribution of nanoparticles can be recovered by summing up tensor products of Chebychev polynomials. In the three-dimensional case it can be done according to [1, Eq. (37)]. In a second step, these results have to be rescaled and deconvolved. The rescaling can be carried out pursuant to ([1, Eq. (35)]). For the deconvolution, several methods were proposed. In [1] two methods SLE- ℓ_1 and SLE- ℓ_2 proved to be suitable. In [2] a neural network model was proposed for this step. In this work, we apply a similar neural network model for the first time on three-dimensional simulated data consisting of volumes of simulated blood vessel-like structures and compare it to the two methods from [1].

II. Methods and materials

The neural network model used in this work is derived from [2] without attention gates and adapted to work with three-dimensional data. The encoding path of the model consists of convolution blocks with two convolution layers, each followed by batch normalization and a ReLU activation function. The kernel size of each convolution layer is $3 \times 3 \times 3$ and a padding by one to maintain the spatial dimension. In the first layer of each convolution block the number of feature maps is increased by 64, which leads to 512 feature maps overall at the end of the encoding path.

Furthermore, the spatial dimension is downsampled by max-pooling with a kernel size of $2 \times 2 \times 2$ and stride of 2 after every convolution block. In the decoding path the spatial dimension is then restored by transposed convolution layers with an output padding of one and the same kernel size and stride as the max pooling of the encoding path. Furthermore, in the decoding path the number of feature maps is reduced to one to fuse the output of the decoding path with the output of a shortcut connection

Table 1: The mean results of the U-Net model, SLE- ℓ_1 , and SLE- ℓ_2 deconvolution on the test dataset.

	U-Net	SLE- ℓ_1	SLE- ℓ_2
MAE	0.014	0.071	0.065
SSIM	0.98	0.62	0.69

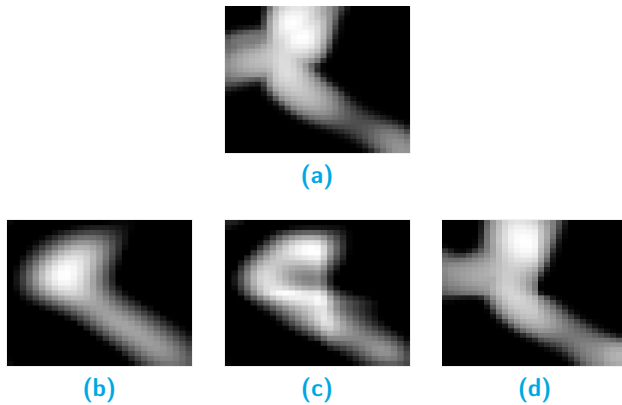


Figure 1: An example slice of the x-y-plane of a volume at $z = 15$. (a) Ground truth volume of the test data set; (b) the solution of SLE- ℓ_1 ; (c) SLE- ℓ_2 ; (d) deconvolution and the prediction of the U-Net model.

to get the final prediction of the model.

II.1. Dataset

To train and test the proposed U-net model, an MPI-scanner simulation was used to generate the required data. The simulation was based on the Langevin model of paramagnetism, excluding relaxation effects, with an Lissajous-type excitation pattern with frequency ratios $f_x/f_y = 33/32$ and $f_x/f_z = 33/34$. The temperature was set to 293 K and a core size of the nanoparticles to 20 nm. Using this setting, 15000 voltage signals were simulated corresponding to 15000 different blood vessel-like structures within a $31 \times 31 \times 31$ volume. From these signals, 15000×3 volumes of size $29 \times 29 \times 29$ were reconstructed using the DCR from [1]. For every signal, three volumes are reconstructed with the DCR, one for each receive coil. The data set then was split into a training set containing 10000×3 samples, a validation set containing 1000×3 samples and test set of 4000×3 samples. The network was trained via the Adam optimizer using a learning rate of 10^{-4} , which was decayed by 0.9 after every 20-th epoch and the mean absolute error as the objective function. The training lasted 146 epochs till no improvement of the objective function was observed. To ensure the neural network did not overfit the training data were randomly

divided into batches of 20 samples in every epoch. Furthermore, the vessel images of the test data were generated with different parameter sets than the training data to assess the generalization capability of the neural

network.

III. Results and discussion

We compared our neural network approach with two deconvolution methods SLE- ℓ_1 and SLE- ℓ_2 from [1]. The regularization parameter λ for SLE- ℓ_1 and SLE- ℓ_2 was selected in order to obtain the lowest possible mean absolute error (MAE) on the test data, which led to $\lambda = 1.79 \cdot 10^{-10}$ for SLE- ℓ_1 and $\lambda = 7.16$ for SLE- ℓ_2 . As a second error measure the structural similarity index (SSIM) was calculated. The means of both error metrics for all three methods are shown in Table 1. The U-Net model achieves the best results out of all three methods. This is also reflected in the visual quality of the reconstructions, as it is shown in Fig. 1, which shows a slice of a volume from the test data set and the corresponding reconstructions of SLE- ℓ_1 , SLE- ℓ_2 and the U-Net model.

IV. Conclusion

In this work we present the deconvolution of reconstructed three-dimensional volumes containing blood vessel like structures. For this, the two deconvolution methods from [1] were compared with our U-Net model, which was derived from [2]. The U-Net was able to achieve the best reconstruction results on a simulated data set. However, further tests on real data and data with noise are necessary to better assess the capabilities of the proposed approach.

Author's statement

Conflict of interest: Authors state no conflict of interest.

References

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